Chance News (May-June 2005)

From ChanceWiki

> We've heard that a million monkeys at a million keyboards could produce the complete works of Shakespeare. Now, thanks to the Internet, we know that is not true. ~ Robert Wilensky

Table of contents

Forsooth

As the stakes increase, Prime-Number theory Moves Closer to Proof
Wall Street Journal, Science Journal, April 8. 2005, B1
Sharon Begley

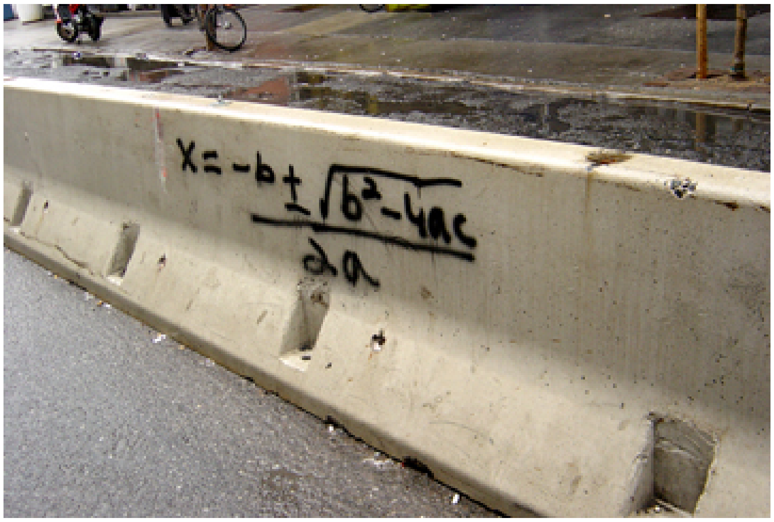
Follow the points to find a Super Bowl champ
New York Times, 223 January, 2005, p 11
Aaron Schatz

> The explanation rests in a mathematical formula created by the baseball analyst Bill James and introduced in the 1980 Baseball Abstract. James determined that the record of a baseball team could be approximated by taking the square of team runs scored and dividing it by the square of team runs scored plus the square of team runs allowed. **Because of its similarity to the geometric method for determining the sum of the angles in a right triangle, he called it the Pythagorean theorem**.

DISCUSSION QUESTION:

How close is the Pythagorean theorem to the theorem that the sum of the angles in a triangle is 180 degrees?

P.S. Norton Star provided this picture observed by a student Tosin while walking in New York. Evidently New Yorkers are determined to not forget the quadradic formula!



Case leads to fight on Jewish representation on juries

This item was suggested by Peter Kostelec.

Case stirs fight on Jews, juries and execution
The New York Times, March 16, 2005
Dean E. Murphy

John R. Quatman was a prosecutor for 26 years in Alameda County California and is now a lawyer in Montana.

In 1987 Quatman was the prosecutor when Fred Freeman was found guilty of murder and robbery at a bar in Berkeley. The jury recommended the death penalty and Freeman was put on San Quentin's Death Row. He is now seeking to appeal his conviction.

Quatman has provided a habeas corpus petition (a petition typically used to appeal state criminal convictions to the federal courts when the petitioner believes his constitutional rights were violated by state procedure) stating that at the 1987 trial the late Judge Stanley Golde, during the jury selection, advised Quatman that no Jew would vote to send a defendant to the gas chamber. In his petition, Quatman said that Golde helped him keep Jews off the jury. And Quatman recommended the death penalty for Freeman.

Quatman claimed that it was standard practice to exclude Jewish jurors in death sentences and this practice extended to African-American women, though this was not a problem for the Freeman trial. In rulings going back to 1880 the United States Supreme Court has ruled that it is illegal to reject jurors on the basis of race, and the California Supreme court in 1978 extended that prohibition to religion.

On Tuesday the California Supreme Court will investigate Quatman's sworn declaration. If they find Quatman's claims are credible Freeman will likely get a new trial.

Quatman's declaration is being used in the appeal of another Alameda inmate, Mark Schmeck. The Habeas Corpus Resource Center (HCRC) provides counsel to represent indigent (poor) men and women under sentence of death in California.The HCRC is representing Freeman in his appeal. Working with Schmeck's lawyers, the HCRC reviewed the jury selection in 25 capital trials in Alameda from 1984 to 1994.

The review found that 12 people who identified themselves as Jews were called to the jury box and the prosecution rejected all 12. They also found that of the 17 who had surnames judged to be Jewish names, the prosecution rejected 15. Overall they found that non-Jews were excluded at a rate of 49.97% and Jews and those with Jewish surnames were excluded at a rate of 93.10%.

Cliff Gardner, a lawyer for Mr. Schmeck, said that the statistics from the 10-year review of the capital trials spoke for themselves. Mathematician Phillip Farmer said that the probability of randomly striking 27 of 29 Jews is less than 1 in 1.6 million.

DISCUSSION QUESTIONS:

(1) How do you think Farmer got his 1 in 1.6 million probability?

(2) What problems are there in trying to estimate the probability that an event in the past occurred?

(3) After this was written, it was reported in the New York Times (April 6, 2005) that Judge Kevin Murphy, of Santa Clara County Superior Court, concluded that Quatman lied when he said an Alameda County judge encouraged him to exclude Jews from a jury in the trial of a man sentenced to death in 1987. The article says that Judge Murphy's opinion will be forwarded to the Supreme Court for a final ruling.

Evidently, Murphy's conclusion was reached on the basis of interviews with Quatman and others involved in the case. The statistical evidence seems not to have played a role in his decision. Do you think it should have?

Vermont pays heavy war burden

The price they paid: By several measures, Vermont bears heavy war burden.
Valley News, January 30, 2005
Jodie Tillman.

The *Valley News* is the local paper covering a region in New Hampshire and Vermont that includes Dartmouth College. Their writers often consult Dartmouth faculty. For this article, the writer Jodie Tillman consulted Greg Leibon from the Dartmouth Mathematics Department.

Tillman obtained data to see if Vermont soldiers and Marines deployed to Afghanistan and Iraq are subject to greater risk than from those from other states. She asked Greg to help her analyze the data. The article is available here. Links to her data are at the end of the article. Her data included both deaths per capita and deaths per deployment. We will use only the data related to deaths per deployment. This data set gives, for each state, the number of soldiers and Marines deployed to Afghanistan or Iraq from the beginning of the Iraq war on March 2003 to Oct. 31, 2004. This data, with the computations we use, is available here. We will discuss some of Greg's analysis but we encourage readers to also read his more complete analysis. His analysis can be found here.

From the data, we see that Vermont had 1,613 soldiers and Marines deployed in the period under consideration and had 9 casualties during this period, giving it the highest death rate for all the states. Let's consider how we might design a test to determine if the high death rate in Vermont is just bad luck. For this test the null hypothesis is that the causalities are independent and the probability that a particular soldier or a Marine is killed is the same for all those deployed. With this null hypotheses, the number of casualties in a particular state has a binomial distribution $B(n,p)$ with n the number deployed from the state and p the proportion of casualties among those deployed in all the states.

Greg calls this the *naive* test. This is because it would be equally newsworthy if any other state had an apparent unusually high death rate. So we now consider a test to see if at least one of the 50 states has more casualties than could be explained by chance. For our first attempt we use the same null hypothesis and do a test for each state just the way we did for Vermont. Then we reject the null hypothesis if any of the individual states, tested as our previous test for Vermont, would reject the null hypothesis.

But if we do that, and the null hypothesis is true, the probability that we reject the null hypothesis is $(1 - (1 - .05))^{50}$

= .92 which makes this a ridiculous test. A more reasonable procedure is to choose a lower confidence level for each state and choose this so that the confidence level for the overall test is .05. For this we need to choose the confidence level α for the individual states to satisfy the equation $(1 - (1 - \alpha)^{50}) = .05$. Asking Mathematica to solve this we obtain $\alpha = .00102534$. Thus we will choose the confidence level for each state to be .001.

We have seen that, under the null hypothesis, the probability that Vermont has 9 or more casualties is .0033, so this test does not lead to rejecting the null hypotheses. Consider now Massachusetts, the state with the second highest death rate. Massachusetts had 7146 deployed and 28 casualties. Making the same kind of computation we did for Vermont, we find that, under the null hypotheses, the probability that Massachusetts has 28 or more casualties is .0002. This is less than our confidence level .001, so for this more general test we can also reject the null hypothesis. Incidentally, one occasionally sees a medical study, for example a study to test if a new drug is more effective than placebo, that starts off with a single test and a 5% confidence level, and along the way the researchers find other tests that can be used to test the effectiveness of the drug. They then report the drug to be effective if any of the individual tests reject the null hypothesis without changing the confidence level. As we have seen, this can give them a much better chance of rejecting the null hypotheses (showing the drug is effective) when in fact this is not the case. We might think we have shown that we cannot explain the death rates as the result of chance. But Greg also points out that the assumption in the null hypothesis that the casualties are independent is probably not a good assumption since, for example, there might be incidences where several soldiers are killed all of whom are from the same National Guard unit and hence from the same state. In his Commentary Greg discusses models that can take this into account. This is an interesting discussion and we encourage our readers to read this in his [http//www.dartmouth.edu/~chance/ForWiki/GregComentary.pdf] Commentary.

Of course the *Valley News* article did not include any of this technical stuff. We read:

Gregory Leibon, a visiting professor in Dartmouth College's mathematics department who reviewed the *Valley News* findings, said the numbers of soldiers killed or injured is too small to draw broad conclusions, including whether Vermont soldiers are more likely to die. He noted that the addition or subtraction of a few deaths or injuries could change rankings.

> "On statistical grounds, you could not reject the notion that it's not just bad luck, said Leibon".

DISCUSSION QUESTIONS:

(1) What does this last line really say? How do you think readers interpreted this statement? Do you think Greg was quoted correctly?

(2) Looking at the data we see that Florida had 62572 deployed in the period considered and only 54 casualities. The expected number of casualities under the null hypothesis is 113.87. We note that 54 casualities is 5.62 standard deviations below the expected value. Mathematica tells us that, under the null hypothesis, the probabiity of 54 or fewer casualities is 2.924099646x10^(-10). What do we make of that?

(3) A study carried out by Robert Cushing and reported by Bob Bishop in the Austin American Statesman, 12 October 2003, showed that the rural populations had a higher death rate per capita than those in urban populations. How might this be explained?

Seven statistical cliches used by baseball anounncers

In a game of Statistics, Some Numbers Have Little Meaning
New York Times, April 3, 2005, Section 8, Pg 10
Alan Schwarz

The author writes: with statistics courtesy of Stats Inc., the following is a user's guide to the facts behind seven statistical cliches. We have included excerpts from his explanation and recommend reading his complete discussions.

(1) HAS A 75-6 RECORD WHEN LEADING AFTER EIGHT INNINGS

Teams leading after eight innings last year won about 95 percent of the time (translating to a 77-4 record in 81 games); that 75-6 record would be two full games worse than average. Even after seven innings, teams with leads typically win 90.1 percent of the time.

(2) HOLDS LEFTIES TO A .248 AVERAGE

Middle relievers have become ever more important in baseball, particularly left-handed specialists who jog in to face only one or two left-handed hitters. Last year, left-handed middle relievers held fellow lefties to a .249 collective average, 18 points lower than the major league-wide .267 average in all other situations. Someone yielding a .248 average sounds good but is merely doing his job.

(3) HAS HIT 9 OF HIS LAST 12 GAMES

Last year, each game's starting position players finished with at least one hit 67.1 percent of the time. So across any 12-game stretch, simple randomness will have almost half of them hitting safely in eight or nine games. More than half will wind up with hits in eight or more.

(4) HAS 31 SAVES IN 38 OPPORTNITIES

Relievers who were considered closers converted saves 84.8 percent of the time last season -- 32 times for every 38 chances.

(5) HAS STOLEN 19 BASES IN 27 ATTEMPTS (70%)

Players batting first and second in their lineups, usually speedy table-setters, stole bases 73.7 percent of the time last season.

(6) LEADS N.L. ROOKIES WITH A .287 AVERAGE

Interesting, perhaps, but most people do not realize how few rookies play enough to be considered for this type of list. Last year, six rookies reached the standard cutoff of 502 plate appearances to qualify for the batting title.

(7) HITS .342 ON THE FIRST PITCH

The stat line many people use to make these claims reads "on 0-0 counts". What people do not realize is that on "0-0 counts" includes only at-bats that end on the first pitch; in other words, the hitter put the ball in play. Removing every time a hitter swings through a pitch or fouls it off will make anyone look good.

Numbed by the numbers, when they just don't add up

Numbed by the numbers, when they just don't add up
New York Times, 23 January 2005, The public editor
Daniel Okrent

The public editor column appears twice monthly. The present commentary focuses on "complaints...about innumeracy at The Times."

It is easy for jounalists to uncritically accept numerical figures provided by an outside source. For example, in November 2004, a study by the New York City Comptroller's office asserted that New Yorkers spend more than $23 billion annually on counterfeit goods. This translates to a nonsensical $8000 per household, but apparently no one at the Times tried this arithemetic before running the story. Many other examples are presented.

See the discussion of this article for some other interesting examples.

Dan Rockmore's book: Stalking the Riemann Hypothesis

Stalking the Riemann Hypothesis
Pantheon Books, New York, 2005
Dan Rockmore

The Proof: an interview with Dan Rockmore
New Hampshire Public April 12. 2005
John Walters

As the stakes increase, Prime-Number theory Moves Closer to Proof
Wall Street Journal, Science Journal, April 8. 2005
Sharon Begley

Math Monster
The Telegraph (Calcutta, India), April 8, 2005
Pathik Guha

In 1998 the Mathematical Sciences Research Institute in Berkeley, California had a three-day conference on "Mathematics and the Media". The purpose of this conference was to bring together science writers and mathematicians to discuss ways to better inform the public about mathematics and new discoveries in mathematics. As part of the conference, they asked Peter Sarnak, from Princeton University, to talk about new results in mathematics that he felt the science writers might like to write about. He chose as his topic "The Riemann Hypothesis" This is generally considered the most famous unsolved problem in mathematics and is the major focus of Sarnak's research.

In his talk, Sarnak described some fascinating new connections between the Riemann Hypothesis, physics and random matrices. He used only mathematics that one would meet in calculus and linear algebra. Sarnak's lecture, and a discussion of his talk by the science writers, can be found here under "Mathematics for the Media"

Unfortunately, Sarnak's talk was not fully appreciated by the science writers. The first writer to comment said that she felt like she did when she was in Germany and a friend took her to a party. At the party the Germans initially tried to speak to her in English, and she could understand them pretty well, but then they would drift into German and she was lost.

Another science writer said that, to write an article based on Sarnac's talk, she would have to sit down with him and have him explain what he had said in a way she could understand. Then she would have to write an article for her readers without using formulas or mathematical symbols--a few graphics would be o.k.

With his book "Stalking the Riemann Hypothesis", Dan discusses the Riemann Hypothesis and its history in a way that the science writers proposed for the general public. He does this by explaining the relevant mathematical concepts in terms of concepts familiar to his readers. For example the rate of increase is discussed first in terms of the spread of a rumor and the logarithm in terms of the Richter scale. Density is described in terms of population density noting that the average number of people per square mile living in South Dakota is quite different from that of New Jersey. Then we read

> Similarly, we can ask how many prime numbers "live in the Neighborhoods" of a particular number? Gauss's estimates imply that as we traipse along the number line with basket in hand, picking up primes, we will eventually acquire them at a rate approaching the reciprocal of the logarithm of the position that we've just passed.

While there are no formulas and minimal math symbols, Dan does make good usage of great graphics made by our mutual friend Peter Kostelec.

Along the way, Dan gives a lively discussion of the great mathematicians Euler, Gauss, Riemann and many others up to present working on solving the Riemann Hypothesis. He also gives the readers an understanding of what mathematics and mathematical research is all about. In the *Telegraph* Dan is quoted as saying:

> My overarching goal has always been to provide a window through which the public might get some sense of what mathematical research is like while showing the surprising breadth of math as a subject, I felt that the best way to achieve this was through a single story, for I believe that narrative is the key to holding a reader's interest. It seemed to me that the history of the Riemann Hypothesis - given its central place in modern mathematical history and research - would provide such an opportunity. The story of the search for its resolution would provide a structure off which I could hang a broader story of modern math.

This is not Dan's first attempt to give the general public a better understanding of what mathematicians do, what mathematics is all about and how it effects our daily life. (I once mentioned to a musician that I was a mathematician and he said, "I guess you learn how to multiply and add numbers faster than we can".)

Dan's first article for the general public was an essay in the *New York Times* in 1998. In this essay Dan used the movie" Good Will Hunting" to illustrate stereotypes of mathematicians. He writes:

> The main messages of the movie are old and trite: You are either someone who can do math or you are not; mathematics is impossible to explain to others, even other mathematicians, and to be a mathematician and to think about mathematics is to separate yourself from most of society. After all, what could be more at odds with Will's working-class, street-wise background than a talent for mathematics?

Dan then gives examples from the arts to show the pervasiveness of mathematics in the real world. For example, the Movie "Sliding Doors" illustrates how long-term behavior of a system can be highly dependent on the initial conditions -- the butterfly effect. He remarks that identifying these connections does not need to lead to madness as it does for the hero of the movie Pi.

More recently Dan has presented on Vermont Public Radio a series of mathematical commentaries with jazzy titles such as "Math, a love story", "Halving Your Cake" and "Can you hear the shape of your date?" giving explanations of important modern mathematical results that we see in everyday life. You can hear these commentaries here.

Last year Dan and his colleagues Wendy Conquest and Bob Drake made a movie called "The Math Life" in which leading mathematicians talk about how they got into mathematics, what kind of mathematics interests them and what it is like to work on and solve a mathematical problem. This movie was shown on public television and used in numerous classrooms.

"Stalking the Riemann Hypothesis" is Dan's greatest challenge to bring mathematics to the general public. It is hard to image a better mathematical story to show the general public the beauty, the excitement, and the importance of mathematics. Dan is an engaging writer and he tells the story in a wonderful way. We can hardly wait for the movie!

The powerball lottery suspects fraud, but it's the Fortune Cookies

Who needs Giacomo? Bet on the fortune cookie

New York Times, May 11, 2005, National desk; Pg 1
Jennifer Lee

This article reports unexpected winnings in the Powerball lottery of March 30, 2005 which lottery officials thought might be fraudulent but which had a much simpler explanation.

For the Powerball lottery, a player chooses 5 distinct numbers from 1 to 53, which we will call the "basic numbers." In addition, the player chooses another number between 1 and 42 which we will call the "bonus number." The lottery randomly chooses 5 basic numbers and one bonus number. If your 6 numbers agree with those chosen by the lottery you win the jackpot (a huge amount). If there is more than one jackpot winner, you share the jackpot with the other winners. There are 8 additional prizes which you do not have to share with other winners. For example, if you buy a $1 ticket and your 5 basic numbers match those of the lottery but your bonus number does not, you win $100,000.

When you buy a $1 lottery ticket, for an additional $1, the lottery offers another bet called the "Power Play". For the Power Play, the lottery randomly chooses a number from the numbers 2,3,4,5,5. This number is called the "multiplier". If you make this bet and you win any prize other than the Jackpot, the prize is multiplied by the multiplier.

On March 30 drawing of the Powerball lottery, 110 players made a $1 bet, choosing as their five basic numbers 22,28,32,33,39 and as their bonus number 40. The lottery chose the same five basic numbers but chose 42 for their bonus number. The lottery chose 5 for the multiplier. 89 of the 110 winners did not choose the Power Play and so each won $100,000. 21 players chose the Power Play and, since the multiplier was 5, they each won $500,000 dollars. Thus the lottery paid out 19.4 million dollars to these winners. Actually, they didn't have to pay out this much since on the back of a ticket:, in small print, we read:

> In unusual circumstances, the set prize amount may be paid on a pari-mutual basis, which will be lower than the published prize amounts.

Evidently the Lottery officials decided not to use this option in this case. In addition the article states that the Lottery keeps a $25 million reserve for odd situations.

Power ball officials stated that, considering the number of tickets sold in the 29 states, they expected 4 or 5 winners. The article quotes Chuck Strutt, executive director of the Multi-State Lottery Association as saying:

> Panic began at 11:30 pm. March 30 when he got a call from a worried staff member. We didn't sleep a lot that night. Is there someone trying to cheat the system?

The lottery athorities tried a number of theories about how people choose their numbers. For example many players pick their numbers following a geometric design on the ticket. Nothing worked. But then the first three winners said that they had obtained the numbers from a fortune cookie. With this lead, they just had to find the fortune cookie maker who had the winning numbers. They found that many different brands of fortune cookies come from the same Long Island City factory owned by Wonton Food. This company turns out four million fortune cookes a day, which are delivered to dealers over the entire country. When shown the numbers, Derrick Wong, of Wonton Food, verified that they had used these numbers. The numbers were chosen from a bowl but the company plans to switch to having them chosen by a computer and Derrick plans to start playing the lottery.

DISCUSSION QUESTIONS:

(1) The articles says:

> Of course, it could have been worse. The 110 had picked the wrong sixth number -- 40, not 42 -- and would have been first-place winners if they had.

Worse for whom?

(2) How do you calculate the probability of getting the 5 basic numbers but not the bonus number correct?

(3) Do you think that the sales of fortune cookies will increase?

Red enhances human performance in contests

Red enhances human performance in contests
Nature, Vol. 435, May 19, 2005
Russell A. Hill, Robert A. Barton

Hill and Barton examined the results of the 2004 Olympic games in four categories of competition—boxing, tae kwon do, Greco-Roman wrestling, and freestyle wrestling—chosen because in each match, one contestant wears red, the other blue. Within weight classes, color assignments are apparently made randomly in the first official round of competition; subsequent assignments are then determined by the initial roster. (In boxing, for example, the winner of bout 1 plays in red in the second round against the winner of bout 2, who plays in blue. The same arrangement is used for the winners of bouts 3 and 4, 5 and 6, and so on.) The authors found that for each sport, significantly greater than 50% of bout winners wore red outfits.

To study the findings more closely, Hill and Barton focused on competitions in which the two contestants were most evenly matched. According to the article, they did this because "wearing red presumably tips the balance between losing and winning only when other factors are fairly equal." Such matches do appear to represent the only cases in which there were significantly more red than blue winners. (A description of their methods can be found in a supplementary text file at Nature's website—see below.)

The authors, both members of the Evolutionary Anthropology Research Group at the University of Durham in the UK, evidently favor a behavioral/biological explanation for the apparent red advantage: "Red coloration is a sexually selected, testosterone-dependent signal of male quality in a variety of animals, and in some non-human species a male's dominance can be experimentally increased by attaching artificial red stimuli. Here we show that a similar effect can influence the outcome of physical contests in humans."

The article, excel data file, and supplementary text file are available at Nature's website. More detailed information about the competitions can be found at the 2004 Olympic Games website.

DISCUSSION QUESTIONS :

(1) Assuming that Hill and Barton's results are valid, what do you think is the most likely explanation for the surplus of red winners?

(2) The authors don't discuss the possibility that the color of competitors' outfits affects the performance of the contest judges. (Judges assign points to each athlete during the match.) How might you evaluate this possibility?

(3) Get the data and carry out your own test to see if there is a signficant difference between the reds and the blues.
Red enhances human performance in contests

Mix math and medicine and create confusion

Mix math and medicine and create confusion
New York Times, April 26, 2005, F 11
Richard Friedman, M.D

This article provides an interesting exchange between a doctor (Doctor Friedman) and a statistician (Judith Singer). We give the entire exchange as presented in this article by Dr. Friedman.

> Patients may not know it, but there are two questions that make doctors cringe. The most common is, If you were me, which treatment option would you pick? The tougher one is, What are the chances that this treatment will help me?' Both questions cut to the heart of medical decision making and involve

assessing risk and probability, which does not come naturally to many people.

For example, a depressed patient told me she had read that the chances were 60 percent that she would respond to the antidepressant I had prescribed for her.

"That means that 60 percent of the time I will feel better on this, right?" she asked.

Well, not exactly. I explained that if 10 people with a depression just like hers walked into my office, about 6 would be expected to respond to that antidepressant.

But the statistics, I told her, referred to a large sample, not an individual. She would either improve with this treatment or she would not, I said, but she shouldn't worry because we would keep trying until we found a treatment that worked.

"You mean my chances of getting better are really only 50 percent?" she asked with dismay.

Dr. Judith D. Singer, a statistician and professor at the Graduate School of Education at Harvard, explained "You and your patient are confusing two different concepts. The number of possible outcomes -- in her case either responding or not responding to an antidepressant -- has nothing to do with the actual probability of either outcome happening."

For example, Dr. Singer said, "Either a woman is pregnant or not. She can't be a little pregnant. But that doesn't mean that she has a 50 percent probability of being pregnant."

A woman who takes a fertility pill may stand a much higher chance of actually getting pregnant than if she goes without it. If my patient was typical of the subjects in the clinical trial she read about, Dr. Singer said, "she is more likely than not to get better on that antidepressant."

DISCUSSION QUESTIONS:

(1) The doctor explained 60% chance of a response from the antidepressant as "If 10 people with a depression just like hers walked into my office, about 6 would be expected to respond to that antidepressant. The statistician's explanation was: If my patient was typical of the subjects in the clinical trial she read about, she is more likely than not to get the better on that antidepressant. What are the pros and cons of these explanations? How would you answer the question?

(2) If your doctor would answer one of the two questions that make doctors cringe, which would you prefer? Why?

Marilyn answers a lottery question

Ask Marilyn
Parade, April 10, 2005
Marilyn vos Savant.

A reader poses the following question:

> My wife and I attended a "reverse raffle," in which everyone bought a number. Numbered balls were then drawn out of a bin one at a time. The last number would be the winner. But when the organizers got down to the last couple of dozen balls, they discovered that some numbered balls had been overlooked. So they added those balls to the bin and continued the drawing. Didn’t the added balls have a much better chance of winning?

Marilyn responds, "Yes, they did. But because everyone had an equal chance of his or her numbered ball being one of those overlooked, the last-minute addition made no difference to anyone’s chance of winning. The raffle was still fair."

Marilyn's answers to probability problems often stir up controversy, and this was no exception. The discussion continues in the column below.

Ask Marilyn.
Parade, June 5, 2005
Marilyn vos Savant.

A reader had this objection to Marilyn's answer:

> Marilyn: I disagree with your answer. Participants whose balls were left out had a higher likelihood of winning. Regardless of whether they had a fair chance of being overlooked, the raffle was not mathematically fair. Assume there were 20 participants. The odds of winning should be one in 20 throughout the game for each contestant. Put 15 balls in a jar and withdraw 10. Then add the missing five. The first 15 balls had a two-in-three chance of "not winning" until the five balls were added. The missing five balls had a zero chance of "not winning" during that time, then had a one-in-10 chance of winning after they were added. Only the five balls that were in the bowl the entire time had a one-in-20 chance of winning.

Marilyn says her original answer was correct, and asks the reader to "consider a scenario in which the added balls were withheld (on purpose) instead of overlooked." She says. "Your explanation works in that case. So it cannot work in the case when the added balls were merely overlooked."

DISCUSSION QUESTIONS:

(1) Do you understand Marilyn's last response?

(2) The reader is actually giving conditional probabilities. Do you agree with their values?

(3) Now consider the reader's set-up, under Marilyn's original assumption that each ball had an equal chance of being overlooked in the first stage. Thus, there are 20 balls, and 15 are initially selected at random and placed in the bin. Now 10 are drawn one at a time at random from the bin. At this point, the 5 balls originally omitted are added to the bin. Then balls are drawn one at a time at random from the bin. Looking at the whole process, does each ball have a one-in-20 chance of being the last ball in the bin?

(4) Upon realizing the correct answer to Marilyn's problem, a chance news reader suggested that this meant that the famous 1970 Vietnam lottery was fair also. Recall that in this lottery 366 balls with the possible dates in a year were put in a bowl and mixed up. Then the balls were drawn out one at a time and the dates on the balls determined the order in which draftees would be called up. It was estimated that those with birthdays were on the last third of the balls drawn would not be called up at all. Å statistical analysis suggested that the balls were not well mixed and as a result those born in the early months were significently more likely to be called up than those born in the later months. But our reader said that, since our birthdays are random, the lottery was still fair. Do you agree?

P.S. You can read the *New York Times* account of the statistical challenge of this lottery here and a nice article by Norton Starr on how to use this lottery in a statistical class here.