Shakin' Things Up: Using the Statistical Approach to Model Natural Disasters

Abstract Earthquakes are one of the natural disasters that Americans have experienced in their life time. Usually the earthquakes that occur in America range from non catastrophic, resulting in minor damages, and even minor expenditures when it comes to restoring the city, or town that the earthquake happened in. But then their are earthquakes with higher than normal magnitudes that are catastrophic, resulting in major damage of the infrastructure within a city or town, as well as high expenditures to reconstruct its infrastructure. Most of the time, the catastrophic earthquakes are severely extreme, but rarely happen in comparison to the earthquakes that happen more frequently than the catastrophic earthquakes. Since earthquakes with high magnitudes happen seldom, how can they be analyzed, or modeled? The Extreme Value Theory is a theory that has been developed in order to model and analyze rare, but extreme events. This paper focuses on using EVT to model the extreme earthquakes within the data of Selected Earthquakes of General Historic Interest, provided by U.S. Geological Survey, and discussing what is EVT and the different components within EVT in relation to the previously mentioned data.

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1 Introduction

The Extreme Value Theory, is becoming a well known discipline in statistics. From this discipline, we are able to focus on extreme and rare events that occur. For this study, EVT will be used to focus on the extreme and rare earthquakes that have happened in the United States from the year 1700, to the year 2011. From the data, we will determine how we can use EVT to analyze and model the data. In this paper, starting in chapter 2, we will discuss the basic principle of EVT which includes the generalized extreme value distribution and generalized Pareto distribution. Also, in chapter 2 we will discuss two methods, block maxima and peak over threshold, which can be used to analyze a data set. Proceeding to chapter 3, we will discuss how two estimation techniques known as maximum likelihood method and method of moments, can be used to calculate the parameters needed for the different distribution types. Lastly, after discussing the estimation techniques in chapter 3, we will finally discuss in chapter 4, how to use the statistical package R and the POT method to process, analyze, and model the data set, Selected Earthquakes of General Historic Interest.

2 The Extreme Value Theory

The Extreme Value Theory (EVT) can be used to analyze data that deviate from the median of probability distributions, and can be used as a tool for analyzing the risk for events that happen seldom, for example events such as the California earthquake of 1906, and Hurricane Katrina from 2005. Within (EVT), there are two different modeling techniques. The first one is the Block

Maxima modeling technique, and the second is the Peak Over Threshold Technique known as (POT). While both are used for modeling large sample sizes for observation, each have more specific uses, especially when deciding which model would be best to work with according to the data set. First we will start with the specifics of the Block Maxima Approach.

2.1 The Block Maxima Method

The Block Maxima Method depends on M_n, where M_n represents the division of observation periods into equal sized, mutually exclusive periods. Within these observation periods, only the maximum observation of each period are chosen. Hence,

$$M_n = max(X_1, ..., X_n),$$

where $X_1, ..., X_n$ are independently identically distributed (i.i.d).M_n follows a Generalized Extreme Value (GEV) distribution.
As $(n \to \infty)$, M_n converges to:

- $$\text{G}(y) = exp\left(-\left[1+\epsilon\left(\frac{y-\mu}{\sigma}\right)\right]^{-1/\xi}\right) \quad if\ \xi \neq 0$$
- $$\text{G}(y) = exp\left(-exp\left(-\frac{y-\mu}{\sigma}\right)\right) \quad if\ \xi = 0$$

GEV has three parameters: the location parameter(μ), scale parameter (σ), and shape parameter(ξ)

2.2 Generalized Extreme Value (GEV)

For GEV, there are three distribution types that can be used, based on the shape parameter. The different types of distributions, according to the condition for ξ, the shape parameter, are as follows:
• $if\ \xi = 0$, the Gumbel distribution is used where

$$\text{G}(x) = exp(-exp[-x])$$

• $if\ \xi = \frac{1}{\alpha>0}$, the Frechet distribution is used where

$$\text{G}(x) = exp\left(-\left[1-\frac{x}{\alpha}\right]^{-\alpha}\right)$$

• $if\ \xi = \frac{-1}{\alpha<0}$, the Weibull distribution is used where

$$\text{G}(x) = exp\left(-\left[1-\frac{x}{\alpha}\right]^{\alpha}\right)$$

Now, that the block maxima method and the generalized extreme value distribution has been discussed briefly, we will extensively discuss the Peak Over Threshold (POT) method, which is the method that has been used for the data used in this study: Selected Earthquakes of General Historic Interest.

2.3 Peaks Over Threshold (POT) Method

POT is used when taking the peak values that occurred during any period of time, from a continuous record. POT focuses on the peak values that exceed a certain threshold. POT depends on u, a very large threshold and is defined by Y_i, where

$$Y_i := X_i - u|X_i > 0$$

are the exceedences over u, which are asymptotically distributed and follow a Generalized Pareto Distribution (GPD).

2.4 The Generalized Pareto Distribution (GPD)

The cumulative distribution function (cdf) of GPD is defined as:

$$F(x) = 1 - \left(1 - \frac{kx}{\alpha}\right)^{\frac{1}{k}}, \; ifk = 0$$

$$= 1 - exp\left(\frac{-x}{\alpha}\right), \; ifk \neq 0$$

which depend on two parameters, the scale parameter (α) and shape parameter(k)

Paralleling the development of extreme value distributions there are three related distributions in the family of generalized Pareto distributions. The distributions are: exponential, Pareto, and beta.

The shape and scale parameter can be estimated by two estimation techniques, maximum likelihood estimation and method of moments.

3 Estimation Techniques to Find Parameters

3.1 Maximum Likelihood Technique

Let $X_1, X_2, ...X_n$, be a random sample, that depends on one or more unknown parameters say, $(\theta_1, \theta_2, ...\theta_m)$. Assume that those unknown parameters $(\theta_1, \theta_2, ...\theta_m)$, are restricted to a given parameter space, Ω. Then the joint probability mass function (p.m.f) of the random sample size $X_1, X_2, ...X_n$, is called the likelihood function, such that

$$L(\theta_1, \theta_2, ...\theta_m) = f(x_1; \theta_1, \theta_2, ...\theta_m) f(x_2; \theta_1, \theta_2, ...\theta_m)......f(x_n; \theta_1, \theta_2, ...\theta_m),$$

where $\theta_1, \theta_2, ..., \theta_m \in \Omega$, and is represented as

$$L(\theta) = \prod_{n=1}^{n} f(x_1; \theta_n),$$

when regarded as a function of $\theta_1, \theta_2, ..., \theta_m$.

Consider

$$[u_1(X_1, X_2, ...X_N), u_2(X_1, X_2, ...X_N), ..., u_m(X_1, X_2, ...X_N)]$$

. We refer to $[u_1(X_1, X_2, ...X_N), u_2(X_1, X_2, ...X_N), ..., u_m(X_1, X_2, ...X_N)]$, as the m-tuple in Ω that maximizes $L(\theta_1, \theta_2, ...\theta_m)$. Then, $\hat{\theta_1} = u_1(X_1, X_2, ...X_N)$, $\hat{\theta_2} = u_1(X_1, X_2, ...X_N)$, ..., $\hat{\theta_m} = u_1 m(X_1, X_2, ...X_N)$, are the unique maximum likelihood estimators of

$$[u_1(X_1, X_2, ...X_N), u_2(X_1, X_2, ...X_N), ..., u_m(X_1, X_2, ...X_N)]$$

.
Now, consider the likelihood function for GPD:

$$L(x_i; k, \alpha) = -n \ln \alpha + \left(\frac{1-k}{k}\right) \sum_{i=1}^{n} \ln \left(1 - \frac{k}{\alpha} x_i\right), for\ k = 0$$

$$= -n \ln \alpha - \frac{1}{\alpha} \sum_{i=1}^{n} x_i, for\ k \neq 0$$

For $k \leq 0$, the range for $\alpha is \alpha < 0 and for$k¿0,$the range for \alpha is \alpha >kx_n$.

3.2 Method of Moments Estimation Technique

Let Let $X_1, X_2, ...X_n$, be a random sample size from a distribution with the following p.d.f:

$$f(x_i; \theta_1, \theta_2, ...\theta_r),$$

where the sample space, $\theta_1, \theta_2, ..., \theta_r \in \Omega$ Then $E(x^k)$, where $k = 1, 2, 3, ...$ is the k^{th} moment of the population, and the sum $M_k = \sum_{i=1}^{n} \frac{x_i^k}{n}$, where $k = 1, 2, 3, ...$ is the k^{th} moment of the sample. We take $E(x^k)$, set it equal to M_k, starting with $k = 1$, and keep equating $E(x^k)$ to M_k until enough equations are provided to find the unique solutions for the parameters say, $\theta_1, \theta_2, ..., \theta_r$. In this study, for GPD, MOM can be used to find the unique solutions for the parameters, say k and α, by finding the first and second population moment, and equating them to each other. To see how this works, consider the p.d.f of the GPD:

$$f(x) = \frac{1}{\alpha} \left(1 - \frac{kx}{\alpha}\right)^{\frac{1}{k}-1}$$

Since there are two unknown parameters k and α, we will need to find the first and second population and sample moments. We will first start with finding the first and second population moments, of the GPD. This can be done by finding the central moment, using the p.d.f of the GPD, to find μ and σ^2 is the second moment, respectively. In order to achieve the task at hand, we will use the expected value approach, instead of the moment generating function (m.g.f) approach, since the mgf of the GPD does not exist for all values of k. So, consider $E\left(1-\frac{kx}{\alpha}\right)^r$, where the rth moment exist when $k > \frac{-1}{r}$. Assume $r = 1$ and $k < 0$, where $0 < x < \infty$. By the definition of expected value, we now have:

$$E\left(1-\frac{kx}{\alpha}\right)^1 = \int_0^\infty \left(1-\frac{kx}{\alpha}\right)^1 \times [\frac{1}{\alpha}\left(1-\frac{kx}{\alpha}\right)^{\frac{1}{k}-1}]dx$$

$$= \frac{1}{\alpha}\int_0^\infty \left(1-\frac{kx}{\alpha}\right)^{\frac{1}{k}} dx$$

using u-substitution:

$$u = 1-\frac{kx}{\alpha}$$

$$du = \frac{-k}{\alpha}dx \;\rightarrow\; \frac{-\alpha}{k}du = dx$$

When $x = 0$, $u = 1$, and when $x = \infty$, $u = \infty$.So,

$$E\left(1-\frac{kx}{\alpha}\right)^1 = \frac{1}{\alpha}\int_1^\infty u^{\frac{1}{k}} du$$

$$= \frac{1}{\alpha}\int_1^\infty \left(\frac{u^{\frac{1}{k}+1}}{\frac{1}{k}+1}\right) \times \left(\frac{-\alpha}{k}\right)$$

$$= \frac{-1}{k} \times \left(\frac{u^{\frac{1}{k}+1}}{\frac{1}{k}+1}\right)|_1^\infty$$

Now we have:

$$E\left(1-\frac{kx}{\alpha}\right)^1 = \frac{-1}{k} \times \left(\frac{u^{\frac{1}{k}+1}}{\frac{1}{k}+1}\right)|_1^\infty$$

$$= \frac{-1}{k} \times \frac{k}{1+k} \times (0-1)$$

$$= \frac{-1}{1+k} \times (-1)$$

$$= \frac{1}{1+k}$$

when $r = 1$, $k < 0$, and $0 < x < \infty$, we have that

$$E\left(1-\frac{kx}{\alpha}\right)^1 = \frac{1}{1+k}$$

Therefore from this, we can conclude that

$$E\left(1-\frac{kx}{\alpha}\right)^r=\frac{1}{1+rk},\ \textit{where}\ 1+rk>0$$

Now that we know $E\left(1-\frac{kx}{\alpha}\right)^1=\frac{1}{1+k}$, we can proceed to find out the first population moment, μ which equals $E(x)$:

$$E\left(1-\frac{kx}{\alpha}\right)^1=\frac{1}{1+k}$$
$$E(1)-E\left(\frac{kx}{\alpha}\right)=\frac{1}{1+k}$$
$$1-\frac{-k}{\alpha}E(x)=\frac{1}{1+k}$$
$$1-\frac{1}{1+k}=\frac{k}{\alpha}E(x)$$
$$\frac{k}{1+k}=\frac{k}{\alpha}E(x)$$
$$\frac{\alpha}{1+k}=E(x)=\mu$$

Next, we need to find our second population moment, σ^2. σ^2 can be calculated by finding $E(x^2)$, hence, $\sigma^2=E(x^2)-[E(x)]^2$. We have already found $E(x)$ now we need to find $E(x^2)$. Remember:

$$E\left(1-\frac{kx}{\alpha}\right)^r=\frac{1}{1+rk}$$

Since we are looking for the second population moment, we can let $r=2$, then

$$E\left(1-\frac{kx}{\alpha}\right)^2=\frac{1}{1+2k}$$

$E(x^2)$ can be found as follows:

$$E\left(1-\frac{kx}{\alpha}\right)^2=\frac{1}{1+2k}$$
$$E\left(1-\frac{2kx}{\alpha}+\frac{k^2x^2}{\alpha^2}\right)=\frac{1}{1+2k}$$
$$E(1)-E\left(\frac{2kx}{\alpha}\right)+E\left(\frac{k^2x^2}{\alpha^2}\right)=\frac{1}{1+2k}$$
$$1-\frac{2k}{\alpha}E(x)+\frac{k^2}{\alpha^2}E(x^2)=\frac{1}{1+2k}$$

$$1 - \frac{2k}{\alpha}\left(\frac{\alpha}{1+k}\right) + \frac{k^2}{\alpha^2}E(x^2) = \frac{1}{1+2k}$$

$$\frac{k^2}{\alpha^2}E(x^2) = \frac{1}{1+2k} + \frac{2k}{k+1} - 1$$

$$\frac{k^2}{\alpha^2}E(x^2) = \frac{(1+k) + 2k(1+2k) - (1+k)(1+2k)}{(1+k)(1+2k)}$$

$$\frac{k^2}{\alpha^2}E(x^2) = \frac{2k + 4k^2 + 1 + k - 1 - 2k - k - 2k^2}{(1+k)(1+2k)}$$

$$\frac{k^2}{\alpha^2}E(x^2) = \frac{2k^2}{(1+k)(1+2k)}$$

$$E(x^2) = \frac{\alpha^2}{k^2} \times \frac{2k^2}{(1+k)(1+2k)}$$

$$E(x^2) = \frac{2\alpha^2}{(1+k)(1+2k)}$$

Remember, $\sigma^2 = E(x^2) - [E(x)]^2$:

$$\sigma^2 = \frac{2\alpha^2}{(1+k)(1+2k)} - [\frac{\alpha}{1+k}]^2$$

$$\sigma^2 = \frac{2\alpha^2}{(1+k)(1+2k)} - \frac{\alpha^2}{(1+k)^2}]$$

$$\sigma^2 = \frac{2\alpha^2(1+k) - \alpha^2(1+2k)}{(1+k)^2(1+2k)}$$

$$\sigma^2 = \frac{2\alpha^2 + 2\alpha^2 k - \alpha^2 - 2\alpha^2 k}{(1+k)^2(1+2k)}$$

$$\sigma^2 = \frac{\alpha^2}{(1+k)^2(1+2k)}$$

Now that we have μ and σ^2, we can now equate our first sample moment with the first population moment as such:

$$\bar{x} = \frac{\alpha}{1+k}\ [\bar{x} = \mu]$$

Where $\bar{x}$ is the first sample moment, and μ is the first population moment.

We can also equate our second sample moment with the second population moment as such:

$$S^2 = \frac{\alpha^2}{(1+k)^2(1+2k)}\ [S^2 = \sigma^2]$$

Where S^2 is the second sample moment, and σ^2 is the second population moment. These two equations, will provide the unique solutions for k and α, in which we will be able to find out the MOM estimators for k and α denoted $\hat{k}$ and $\hat{\alpha}$ respectively. Here are two equations and two unknowns:

$$\bar{x} = \frac{\alpha}{1+k} \textit{ and } S^2 = \frac{\alpha^2}{(1+k)^2(1+2k)}$$

From S^2, we can solve for k, to find $\hat{k}$:

$$S^2 = \frac{\alpha^2}{(1+k)^2(1+2k)}$$

$$S^2 = \frac{\bar{x}^2}{(1+2k)}$$

$$(1+2k) = \frac{\bar{x}^2}{S^2}$$

$$2k = \frac{\bar{x}^2}{S^2} - 1$$

$$k = \frac{1}{2}\left(\frac{\bar{x}^2}{S^2} - 1\right)$$

Now that we know k, we can solve for α, from $\bar{x} = \frac{\alpha}{1+k}$ to find $\hat{\alpha}$:

$$\bar{x} = \frac{\alpha}{1+k};\ k = \frac{1}{2}\left(\frac{\bar{x}^2}{S^2} - 1\right)$$

$$\alpha = \bar{x}(1+k)$$

$$\alpha = \bar{x}\left(1 + \frac{1}{2}(\frac{\bar{x}^2}{S^2} - 1)\right)$$

$$\alpha = \bar{x}\left(1 + \frac{1}{2}\frac{\bar{x}^2}{S^2} - \frac{1}{2}\right)$$

$$\alpha = \frac{1}{2}\bar{x}\left(\frac{\bar{x}^2}{S^2} + 1\right)$$

Our MOM estimators for k and α are:

$$\hat{k} = \frac{1}{2}\left(\frac{\bar{x}^2}{S^2} - 1\right) \textit{ and } \hat{\alpha} = \frac{1}{2}\bar{x}\left(\frac{\bar{x}^2}{S^2} + 1\right)$$

3.3 Calculating $\hat{k}$ and $\hat{\alpha}$ from Actual Data

The calculations needed in R, for the peak over threshold (POT) method was enabled by the POT package downloaded in R. The POT package, is a package in which tools are developed to perform most functions related to EVT, that can be used to statistically analyze POT.Using the statistical package R, the following values for $\bar{x}$ and S^2, where $\bar{x}$ is the mean and S^2 is the variance, were calculated:

```
> mean(MAGNITUDE)
[1] 5.827813
>
> median(MAGNITUDE
[1] 6
>
> sd(MAGNITUDE)
[1] 1.330216
>
> var(MAGNITUDE)
[1] 1.769475
```

Using the equations for $\hat{k}$ and $\hat{\alpha}$, along with the following values for $\bar{x}$ and S^2 calculated in R, we find that:

$$\hat{k} = 4.924 \ and \ \hat{\alpha} = 17.261$$

For this study, both estimation techniques will be used to find the estimation of the shape and scale parameters of the processed data. After comparing techniques, we will be able to see which technique provides a more accurate, unbiased estimator for the shape and scale parameters needed for the Generalized Pareto Distribution.

4 Using POT Method to analyze the extreme cases within Earthquake Data

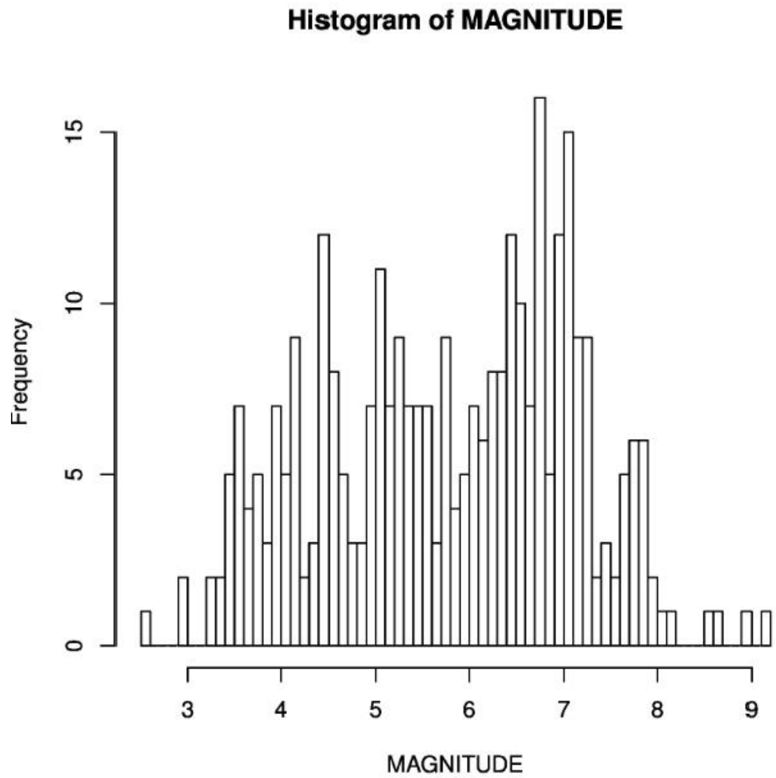
To fulfill a complete understanding of EVT, this study uses a data set of earthquakes. Turning to the United States Geological Survey, the data of Historic Earthquakes in the U.S. and it's Territories were retrieved. This data had an earthquake history from the year 1700 to the year 2011. The sample size of this data set was 320. Since I was working with a large data set, I decided that POT was the best method to use, because I was dealing with earthquake data that occurred during any period of time from the 1700's to 2011, and not all the earthquakes that happened in any one specific year. From the data a threshold can be found, to see when the magnitudes of the earthquakes were really high, resulting in very catastrophic results; the results that would be considered extreme, and of seldom occurrence.

5 Using R to Process Data for Modeling

Using the statistical package R the data was processed so that the following information could be found: 5 number summary, a histogram, box plot, Q-Q plot, mean residual plot and mean excess plot. Before analyzing the graphs with respect to my data set, let's first expound on the significance of each graph.

5.1 The Histogram

A histogram is a way to graphically analyze how the distribution of the processed data set centers itself about the mean, the multiple peaks (modes) within the data, as well as the skewness of the data.



Here we can see the spread of the data. The magnitude of the earthquakes range from 2.0 to more than 9.0. Also, the histogram shows the frequency of the earthquakes that had magnitudes of 2.0 or higher.

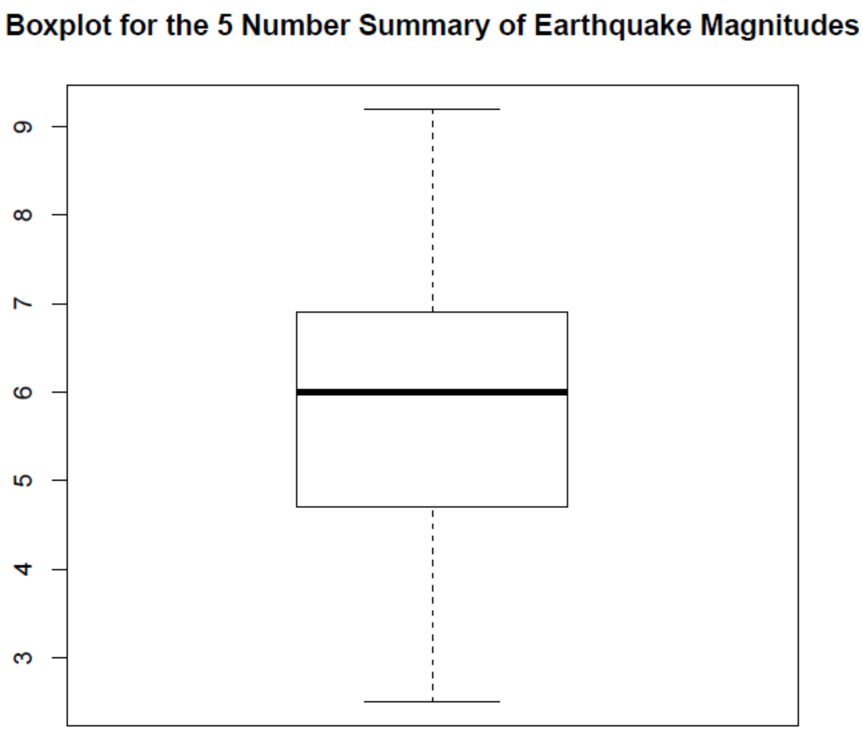
5.2 The Box Plot

The box plot provides a visual for the 5 number summary plot, which includes the median of the data, the 1^{st} and 3^{rd} quartile, and the maximum and minimum values of the data. This is the 5 number summary provided by *R*:

```
summary(MAGNITUDE)
 Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
2.500   4.700   6.000   5.828   6.900   9.200

fivenum(MAGNITUDE)
l] 2.5 4.7 6.0 6.9 9.2
```

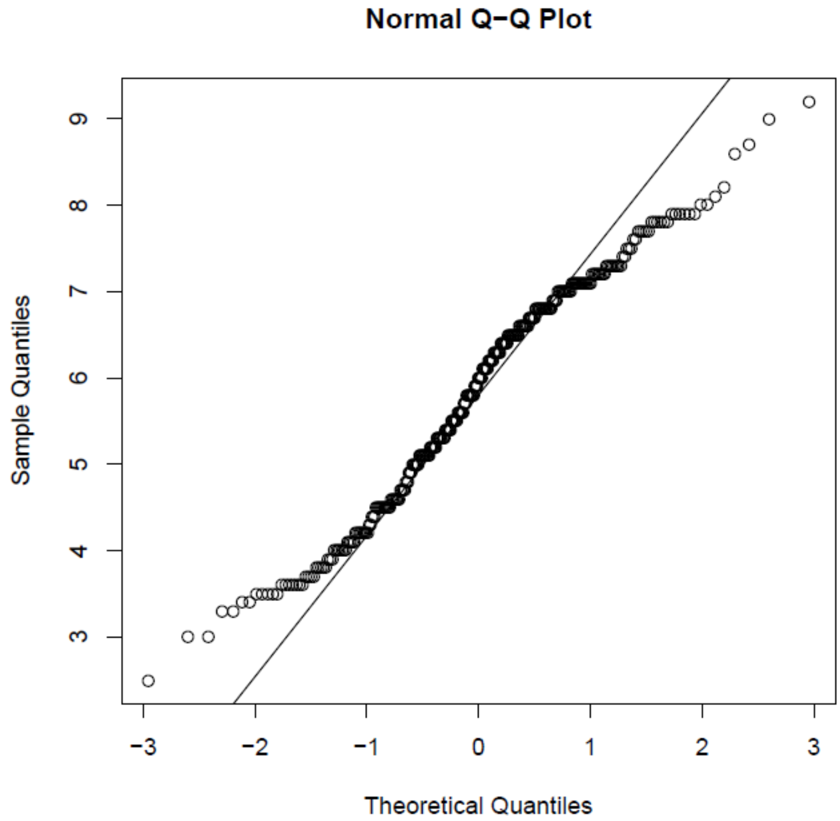
looking at the 5 number summary, let's see how the box plot portrays the summary:



In comparison to the 6 number summary, we see that our maximum value and minimum value of the data is 9.2, and 2.5 respectively which the box plot portrays with the highest bar that is at 9.2, and the lowest bar that is at 2.5. Notice that there is a thick black line within the box. This line is the median of the data. According to the summary provided the median of the data is 6.0, and that is where the line is placed. The space below the black line is the 1^{st} quartile and the space above the black line is the 3^{rd} quartile (which seems to be where most of our data lies).

5.3 The Q-Q Plot

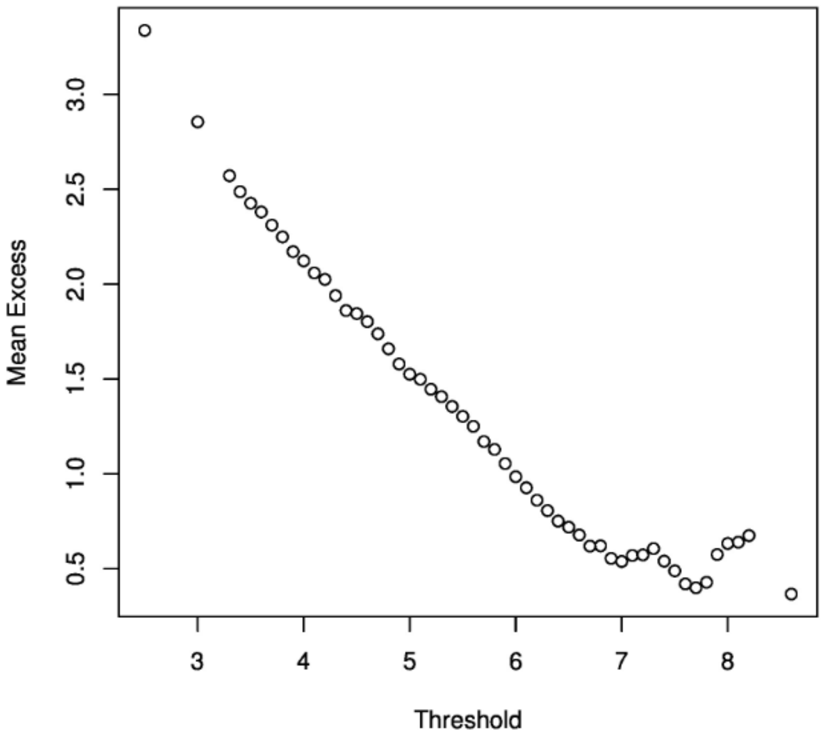
The Q-Q plot is a graphical analysis of the distribution of the processed data compared to the normal distribution. This shows what data follows closely to a normal distribution and which data starts to deviate from a normally distributed pattern.



In the above graph, the line represents a normal distribution, in comparison to the distribution of the processed data. The data that falls on the line, (hence the circles between the theoretical quartiles of -1 and 1) are the data that follow closely to the normal distribution. The circles that start to deviate from the line, are the data that do not follow a normal distribution, the data that we will need for the POT method.

5.4 The Mean Excess and Mean Residual Plots

The Mean Excess and Mean Residual plots, are ways to graphically analyze extreme values to validate a GPD model for the excess distribution, a foundation for POT modeling. Used for estimating and choosing a threshold to work with, when the data begins to deviate from the normal distribution pattern.


Mean Excess
0.5
1.0
1.5
2.0
2.5
3.0
3
4
5
6
7
8
Threshold

For both graphs, you may notice that the data for the threshold values 1-6 the data follows closely to a linear line (normal distribution) until about threshold 7 where the data starts to follow a nonlinear pattern. This is how the threshold of the data, represented by u, was chosen.
Now that the threshold has been chosen to work with, the MLE and MOM estimation techniques can be used. In R, with the data and the threshold, now the shape and scale parameters can be found in order to figure out which distribution within GPD would be a good fit for the data. Before, we end our journey, we first must see, which estimation technique would be better when finding our scale and shape parameters. We want our parameters to be as unbiased as possible. In order to do that, we will use both estimation techniques to find the shape and scale parameters, and then look at them graphically to see which one would give us the most accurate estimation for the parameters needed.

6 Comparing Estimation Techniques using the Threshold u

Now that the the threshold has been analyzed and chosen for the data set, using the package POT in R, we are able to find our shape and scale parameters for the maximum likelihood and method of moments estimation technique.

We are also able to compare how accurate both are as the threshold increases, in the following table:

		Shape (k)		Scale (α)	
u	m	MLE	MOM	MLE	MOM
5.5	184	-.48 (.04)	-1.23 (.20)	1.82 (.14)	2.91 (.35)
6.0	156	-.39 (.05)	-.86 (.15)	1.32 (.12)	1.83 (.23)
6.5	115	-.28 (.06)	-.46 (.12)	.91 (.10)	1.05 (.14)
7.0	65	-.15 (.11)	-.15 (.12)	.62 (.10)	.62 (.11)
7.5	27	-.19 (.18)	-.21 (.19)	.58 (.15)	.59 (.16)

Table 6.1: Notice: m is the number of exceedances over the threshold value u; the standard errors are in parentheses.

While analyzing table 6.1, we are able to recognize two things. One, that our k stays within its restricted bounds $\frac{-1}{2} < k < \frac{1}{2}$, and two, as the value of u increases, the estimators for the scale and shape parameter, begin to take on similar values.

7 Summary and Concluding Remarks

From EVT, the analysis for rare but extreme events, can occur. In order to apply EVT in an effective way when dealing with data of such cases, one must first understand why EVT can be an effective approach, the different components within the extreme value theory, and how to apply the different methods and techniques tot he data accurately, as well as effectively. While exploring the extreme value theory, the statistical package R, and the POT package aids in providing calculations needed with the application of EVT. In this study, we have used the data of Selected Earthquakes of General Selected History, as an application component to illustrate EVT, in conjunction with R and POT. Taking the data we determined that POT was the best approach to analyze the data. Once we decided that POT was a more effective approach than the block maxima approach, we were able to model the data, find a threshold, and talk about the two techniques of estimation, maximum likelihood and method of moments. From these techniques, we were able to find the estimators for the scale and shape parameters, $\hat{\alpha}$ and $\hat{k}$ respectively for the GPD. After finding $\hat{\alpha}$ and $\hat{k}$, we were then able to compare the estimators for MLE and MOM, by increasing the threshold u, to see how unbiased each estimator was. Through comparison, we found that once we reached the threshold values of 7 and 7.5, the estimators of the shape and scale parameters started to become very close in value, and the shape parameter k for both MLE and MOM stayed within the restriction of k, where $\frac{-1}{2} < k < \frac{-1}{2}$. Understanding that no method within EVT is the best method for the analysis of any particular data set, as mentioned before, it is important that EVT is understood for the most effective approach for its methodology. For this study, we were able to discuss both approaches, as well as discuss why POT was a better approach. Since POT worked more effectively when applied to the data, we were able to discuss, apply, and analyze the extreme value theory, with a concentration on the peak over threshold approach, the generalized Pareto distribution, and the estimation techniques maximum likelihood and method of moments.

References

[1] Castillo, Enrique, Hadi, Ali,S.(1997, December). Fitting the Generalized Pareto Distribution to Data. *Journal of the American Statistical Association*, 92(440), 1609-1620.

[2] Ghosh, S. and Resnick, S.(2010) A discussion on mean excess plots. *Stochastic Processes and their Applications*, 120: 1492-1517.

[3] Hogg, R.V., Tanis, E.(2009). . *Probability and Statistical Inference* (8): 273-281.

[4] Hosking, J.R.M, Wallis, J. (1987,August). Parameter and Quantile Estimation for the Generalized Pareto Distribution. *Technometrics*, 29(3), 339-349.

[5] Liu,X(2009) *Data Analysis in Extreme Value Theory.*

[6] McNeil,A.J., Frey, R.,Embrechts,P.(2005). *Quantitative Risk Management.*

[7] Selected earthquakes of general historic interest (2013). In Historic Earthquakes in the United States and Its Territories. *http://earthquake.usgs.gov/earthquakes/states/historical.php/*

[8] R Core Team (2006). A User's Guide to the POT Package. URL http://cran.r-project.org/.