# Using Hunger Games Data to teach Randomization Tests

Kyle A. Caudle

Joint work with: Erica Daniels

Department of Mathematics and Computer Science
South Dakota School of Mines & Technology

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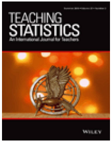

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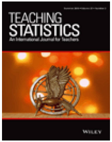
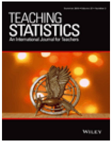


# Background

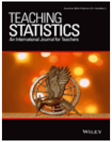
- Work was part of a senior research project by one of our math majors (Erica Daniels)
- Part of the research studied the lottery section of the Hunger Games by Suzanne Collins
- Other research was done on analyzing survival statistics in the arena
- Lottery portion was published in Teaching Statistics (Summer 2015)
- Received the Teaching Statistics Peter Holmes Prize (2015)
- Republished in Stochastik in der Schule (2016)



# Introduction

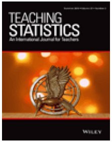
The Hunger Games is an annual event in the fictional country of Panem. Each year, 24 children (tributes) are chosen by lottery from 12 districts to fight to the death in the arena for the entertainment of the Capitol citizens.

We developed a classroom exercise using the fictitious data from the Hunger Games lottery as an example of how to teach students the concept of a permutation goodness of fit test.



# Book Details

- 2 children are chosen from each of the 12 districts.
- 7 (of the 24) children chosen in the lottery were careers (i.e. they were not selected by lottery).
- Katniss volunteered for her 12 year old sister Prim. Therefore, we use Prim's age and not Katniss'.
- A child's name is entered into the lottery once at 12 years of age. If they are not chosen, the next year they receive an additional entry for a total of two. This process continues until the age of 18, resulting in a total of 7 entries.
- A child's chance of selection in the lottery is based on two factors, their age and the number of tesserae they have claimed.

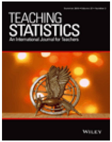


# Lottery Proportions

**Research Question**: Based on these proportions, do we believe the "Game Makers" are following the lottery rules?

Table: Lottery Entries by Age

| Age | Entries | Proportion by Age |
|---|---|---|
| 12 | 1 | 1/28 |
| 13 | 2 | 2/28 |
| 14 | 3 | 3/28 |
| 15 | 4 | 4/28 |
| 16 | 5 | 5/28 |
| 17 | 6 | 6/28 |
| 18 | 7 | 7/28 |

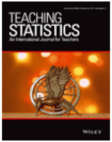


# The Tesserae

A child may receive an extra ration of food for his or her family. In doing so however, they receive one more entry in the lottery. We make the modest assumption that only the older children (Ages 16-18) take an extra ration. One for age 16, another at age 17, etc. Using this modest assumption we arrive at the following proportions.

Table: Lottery Entries by Age

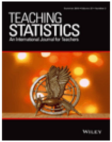
| Age | Entries | Proportion by Age |
|---|---|---|
| 12 | 1 | 1/34 |
| 13 | 2 | 2/34 |
| 14 | 3 | 3/34 |
| 15 | 4 | 4/34 |
| 16 | 5 | 6/34 |
| 17 | 6 | 8/34 |
| 18 | 7 | 10/34 |



# Randomization Tests

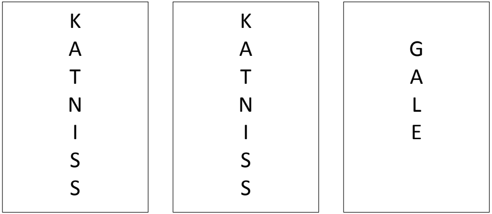
- We introduce a simple randomization test before returning to the more complicated lottery situation.
- Katniss, the heroine in the story, goes squirrel hunting with her friend Gale.
- Katniss kills four squirrels and Gale kills none.
- We would like to test the theory that Katniss is twice as likely to kill a squirrel as Gale.

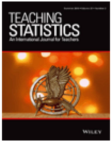
**Research Question**: Is Katniss twice as likely to kill a squirrel as Gale?



# Randomization Tests

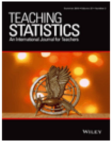
To test the theory, take 3 index cards and write Katniss on two of them and Gale on the third card.



# Randomization Tests

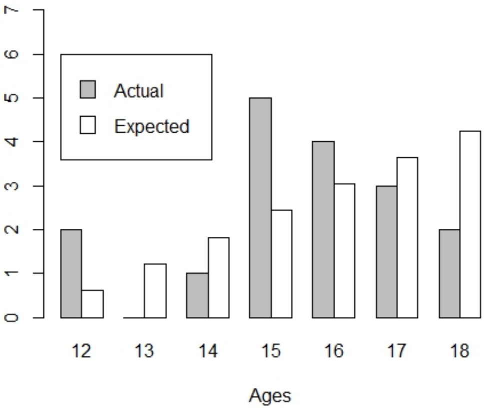
- You can perform this simulation by having a student pick a card to simulate the killing of a squirrel.
- To kill a second squirrel, have the student return the card to the stack so that it is in its original configuration and then redraw.
- There are $3^4 = 81$ possible outcomes.
- Of the 81 outcomes, 16 of those ($2^4$) represent a scenario where Katniss kills all 4 squirrels.
- Pr(4 kills by Katniss and none by Gale) equals 16/81 $\approx$ 20%
- Since this probability is fairly high, we cannot reject the assumption that Katniss kills twice as many squirrels as Gale.

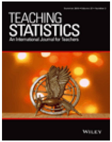


# Permutation Goodness of Fit Test

From the book, we get the actual ages of the 17 children who were selected by lottery. If we multiply the proportions from table 1 by 17 we get the expected counts.

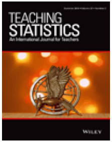
Figure 1: Actual vs. Expected Counts



# Permutation Goodness of Fit Test

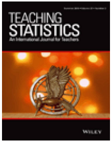
- Since it is not possible to look at all possible outcomes like it was with the squirrel hunting, we perform a randomization test by sampling from the expected distribution (previous slide).
- To simulate this, take 28 index cards and write the number one on one of the cards. Write the number two on two of two of them, etc. Repeat this until the numbers on the cards match the numbers in table 1.
- Next, have students draw a random sample of size 17 from this deck.
- Facilitate a discussion amongst the students regarding how many of each age they should have in the sample of size 17.



# Permutation Goodness of Fit Test

- A standard technique for determining closeness is the chi-square goodness-of-fit test.
- A chi-squared goodness-of-fit test is not permissible here because all but one of the cell counts is less than five.
- We can still use the chi-squared test statistic as a measure of closeness.

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$




$O_i$ is the observed count in categories $i = 1, 2, .., k$

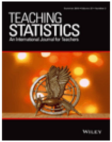
$E_i = np_i$ is the expected number in each category

# Permutation Goodness of Fit Test

$$TS = \frac{(2-0.61)^2}{0.61} + \frac{(0-1.21)^2}{1.21} + \ldots + \frac{(2-4.25)^2}{4.25} = 9.11$$

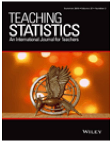
Table: Actual vs. Expected Counts

| Age | Actual | Expected |
|---|---|---|
| 12 | 2 | $17 \times 1/28 \approx 0.61$ |
| 13 | 0 | $17 \times 2/28 \approx 1.21$ |
| 14 | 1 | $17 \times 3/28 \approx 1.82$ |
| 15 | 5 | $17 \times 4/28 \approx 2.43$ |
| 16 | 4 | $17 \times 5/28 \approx 3.04$ |
| 17 | 3 | $17 \times 6/28 \approx 3.64$ |
| 18 | 2 | $17 \times 7/28 \approx 4.25$ |



# Permutation Goodness of Fit Test

- Have several students perform the simulation with the index cards and ask them how their test statistic compares to the test statistic that was calculated using the book data (i.e. 9.11).
- Using an R package that I developed we can run a simulation. Package: Link Manual: Link
- Once installed, the function: rtesthg(m,d) will run a permutation goodness of fit test.
- The function performs a random sample of size 17 from the hunger games distribution and calculates the test statistic. It then repeats the process m times and determines the proportion of times the random sample test statistic exceeds 9.11.




d=1 $\Rightarrow$ no tesserae
d=2 $\Rightarrow$ use tessarae

# Permutation Goodness of Fit Test

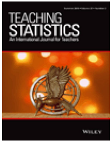
Some examples (No tessare):

rtesthg(10000,1)
0.1589

rtesthg(10000,1)
0.1537

rtesthg(10000,1)
0.156

We see that roughly 15% of the test statistics exceed 9.11.



# Permutation Goodness of Fit Test

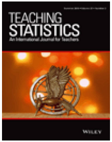
We recalculate the test statistic using expected values using the tessare distribution (table 2). Some examples (with tessare):

rtesthg(10000,2)
0.0528

rtesthg(10000,2)
0.0483

rtesthg(10000,2)
0.0543

We see that roughly 5% of the test statistics exceed 12.55.



# Conclusions

- Randomization tests are a great alternative when the cell count assumptions are not met.
- Computer simulations are an excellent way of helping students understand what would happen if experiments are repeated a large number of times.
- Both items are manageable in an introductory statistics course.
- Combining statistics with 'pop culture' can create interest and make the subject more enjoyable.

**Questions?**