What is the probability you are a Bayesian?

http://www.amstat.org/publications/jse/v22n2/wulff.pdf

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Introduction

- Students in *upper level probability courses* can recite the differences in the Frequentist and Bayesian inferential paradigms, these students often struggle using Bayesian methods when conducting data analysis.

- Specific struggles include translating subjective belief to the specification of a prior distribution and the incorporation of uncertainty in the Bayesian inferential approach.

- Present a hands-on activity involving the Beta-Binomial model to help in understanding the Bayesian approach.

- Albert and Rossman (2009, Ch. 17) - proportion problems
  Kern (2006) - model multinomial probabilities for Pass the Pigs®
  DiPietro (2004) - psychological based project for Bayesian computation

Introduction

Q. *What is the probability you are a Bayesian?*

- Subjective question which differs from the yes-no debate.

- Upper-level statistical inference course consisting of advanced undergraduates and first-year graduates. Students have background in probability theory and integral calculus.

	... In the Bayesian Approach			
	Background	Experience	Preference	Ability
Freddy Frequentist	low	low	low	low
Betty Bayesian	high	high	high	high
Naive Ned	low	low	middle	middle

Assignment

Problem. Let 'Q' denote the question: *"What is the probability that you are Bayesian?"* Answer Q using a Bayesian approach. You have two weeks to complete this assignment.

A. Prior Specification. Let Δ_Q denote the probability that you are a Bayesian. Assign a prior distribution in which $\Delta_Q \sim$ Beta(a, b).

B. Experiment. Conduct an experiment to estimate the true proportion (θ) of 'success' for your randomly selected problem from the collection of N proportion problems.

C. Posterior Distribution. For your scenario, let X take the value 1 if a Bayesian approach was used in B and 0 if a Frequentist approach was used in B. Based upon this single Binomial trial, obtain the posterior distribution of Δ_Q from your prior distribution in A and your observed value of X from B.

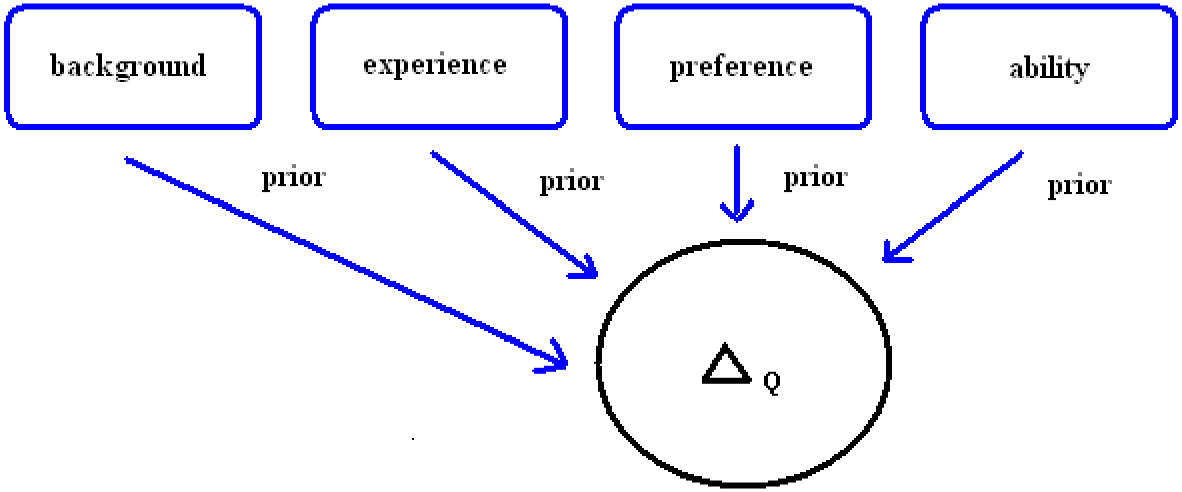
A. Prior Specification

A. Prior Specification. Let Δ_Q denote the probability that you are a Bayesian. Assign a prior distribution in which $\Delta_Q \sim$ Beta(a, b).

A.1. Specify a particular Beta distribution that you feel most accurately describes your affinity towards being a Bayesian. Justify why you chose this particular Beta distribution. A well explained graph could be helpful.

A.2. Give the mean, variance, and quantiles (0.025, 0.25, 0.50, 0.75, 0.95) for your prior distribution.

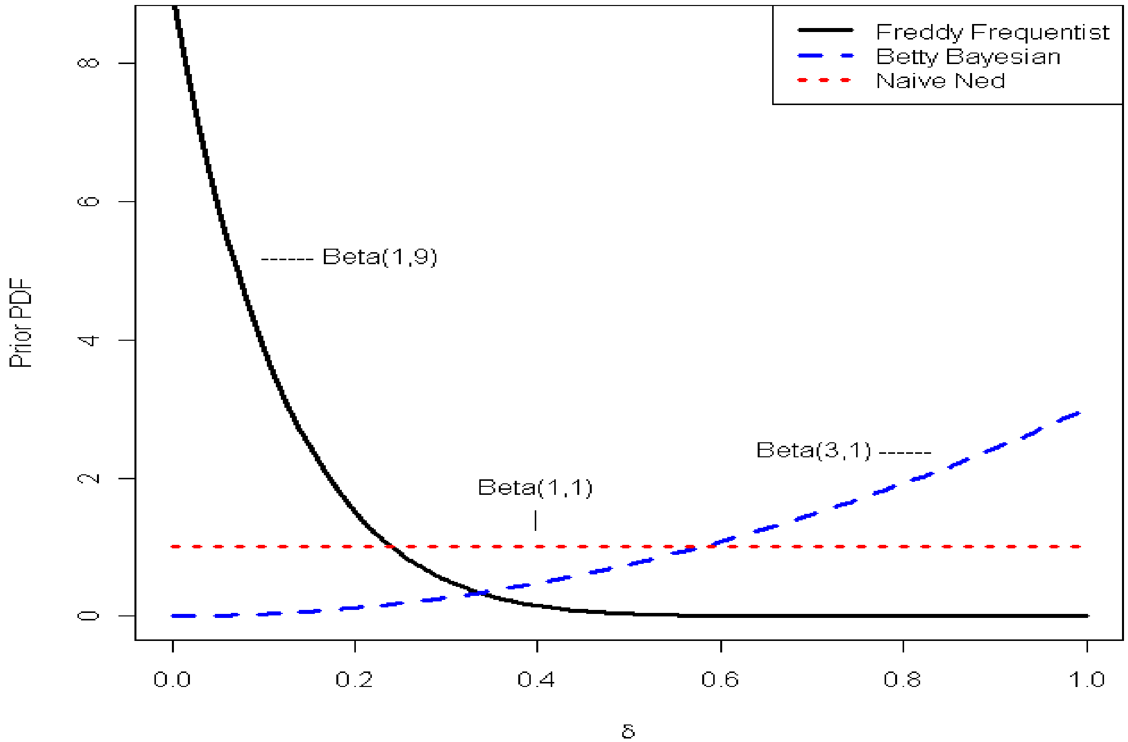
A. Prior Specification



Prior Distribution $\Delta_Q \sim \text{Beta}(a, b)$

Prior PDF $\pi(\delta) \propto \delta^{a-1}(1-\delta)^{b-1}$

A. Prior Specification



Name	a	b	expectation	std dev	quantile 0.25	quantile 0.75
Frequentist Fred	1	9	0.10	0.091	0.032	0.143
Betty Bayesian	3	1	0.75	0.193	0.630	0.909
Naive Ned	1	1	0.50	0.289	0.250	0.750

B. Simple Experiment

B. Experiment. Conduct an experiment to estimate the true proportion (θ) of 'success' for your randomly selected problem from the collection of N proportion problems. Your experiment will consist of n independent trials where Y denotes the number of trials which results in a success.

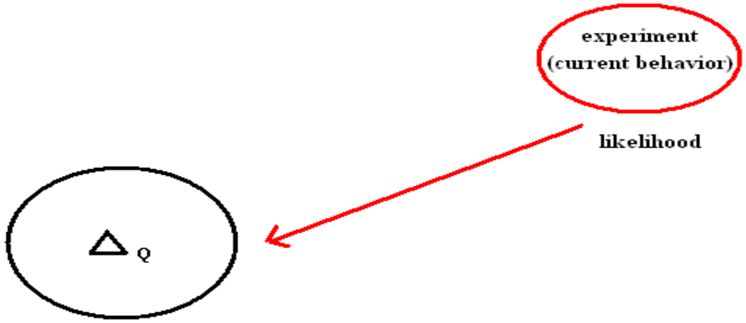
B.1. Obtain a single random number from $1, ..., N$. Give that number to the instructor to receive your randomly selected problem. Briefly describe the problem and any experience that you may have with your problem.

B.2. Carefully explain whether you used a Frequentist or Bayesian inferential approach for the problem in B.1. Your discussion should involve the Bayesian characterizations (B1), (B2), (B3).

B.3. Briefly describe how you obtained sample data for your problem.

B.4. Report the number of successes, point estimate, uncertainty of your point estimate, and the 95% interval estimate for θ.

B. Simple Experiment



- Utilize a collection of N Binomial problems. $Y \sim \text{Binomial}(n, \theta)$
  - Binary outcomes from tossing objects, survey questions, identifications.

- Bayesian Characterizations

(B-1) Parameter of the population model is treated as a random variable.
(B-2) A prior distribution for the parameter is utilized.
(B-3) The inferential procedure is based upon the experimental outcome.

B. Simple Experiment

Frequentist Fred - Assess probability thumbtack lands point up.

Large Sample Frequentist Approach $\quad Y|\theta \stackrel{\bullet}{\sim}$ Normal$(n\theta, n\theta(1-\theta))$

Betty Bayesian - Assess probability that local resident has children.

Bayesian Approach $\quad \Theta|y \sim$ Beta$(a+y, b+n-y)$

Naive Ned - Assess probability that a person in moving vehicle is talking on phone.

Exact Frequentist Approach $\quad Y|\theta \sim$ Binomial(n, θ)

	n	y	$\widehat{\theta}$	$u(\widehat{\theta})$	95% Interval	
Frequentist Fred	50	24	0.480	0.071	0.342	0.538
Betty Bayesian	10	4	0.367	0.120	0.141	0.602
Naive Ned	20	4	0.200	0.089	0.050	0.400

C. The Posterior Distribution

C. Posterior Distribution. For your scenario, let X take be 1 if a Bayesian approach was used in B and 0 if a Frequentist approach was used in B. Then $X|(\Delta_Q = \delta) \sim \text{Binomial}(1, \delta)$ where there is a single experimental trial based upon X. The posterior distribution of Δ_Q is obtained from your prior distribution in A and your observed value of X (x) from B.

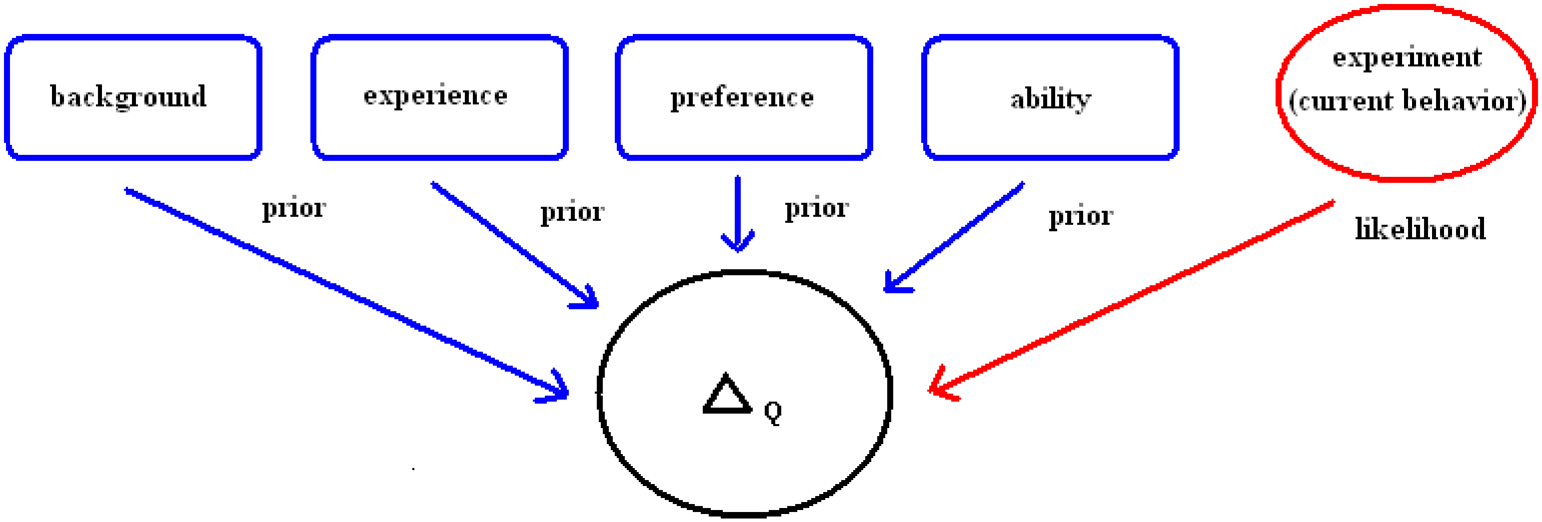
C.1. Give the posterior distribution based upon x . What is the mean, variance, and quantiles (0.025, 0.25, 0.50, 0.75, 0.975) ?

C.2. Provide a plot of your posterior distribution for Δ_Q .

C.3. Describe the change you observed between your prior and posterior distributions on Δ_Q.

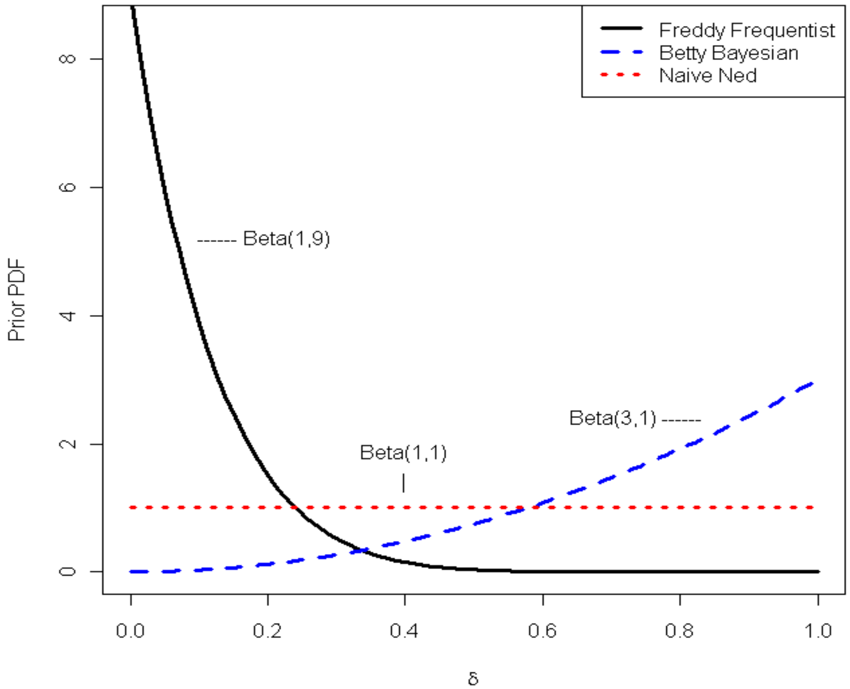
C.4. Give your answer to Q. Include in your answer a description of your posterior estimate and the posterior uncertainty of Δ_Q .

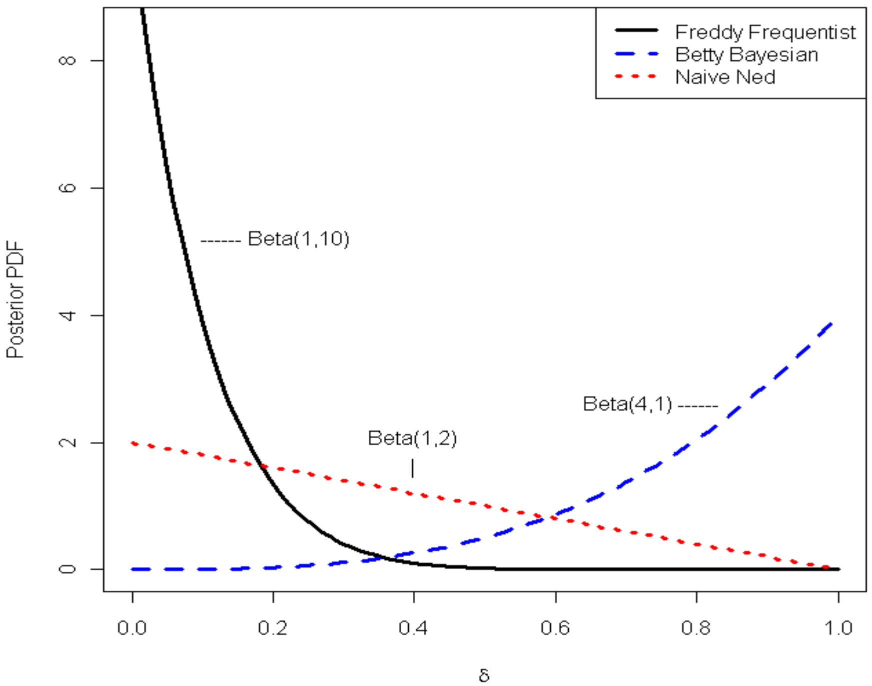
C. The Posterior Distribution



Prior	Likelihood	Posterior
$\Delta_Q \sim \text{Beta}(a, b)$	$X \vert \delta \sim \text{Binomial}(1, \delta)$	$\Delta_Q \vert x \sim \text{Beta}(a + x, n + b - x)$
$\pi(\delta) \propto \delta^{a-1}(1-\delta)^{b-1}$	$f(x \vert \delta) \propto \delta^{x}(1-\delta)^{1-x}$	$\pi(\delta \vert x) \propto \delta^{a+x-1}(1-\delta)^{n+b-x-1}$

C. The Posterior Distribution





Name	a	b	x	expectation	std dev	quantile 0.25	quantile 0.75
Frequentist Fred	1	9	0	0.10 -0.09	0.091 - 0.083	0.032 - 0.028	0.143 - 0.129
Betty Bayesian	3	1	1	0.75 - 0.80	0.193 - 0.163	0.630 - 0.707	0.909 - 0.931
Naive Ned	1	1	0	0.50 - 0.33	0.289 - 0.236	0.250 - 0.134	0.750 - 0.500

Conclusions

Challenges

- A.1. Correctly justifying the prior distribution.
- B.2. Explaining which type of inferential approach was used.
- B.3. Selecting a sample size.
- C.4. Combining all of the information to answer Q.

Assessment

- Delineates distinctions between Frequentist and Bayesian approaches.
- Facilitates deeper discussions on additional Bayesian topics.
- Benefits students with limited Bayesian background by providing opportunity to carry out a simple and practical Bayesian analysis.