# Bootstrapping and randomization: Seeing all the moving parts

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**Causeweb Teaching & Learning Webinar 22 Nov 2011**

# Abstract


- This webinar is a short visual, narrative journey from the vibrating boxplot imagery of the author's 2009 USCOTS Plenary and Wild et al. (2011) to visualisations of bootstrap confidence intervals and, if time permits, randomisation tests. Software under development will be used and made available as-is to any brave souls willing to live on the edge


*References:*


- http://www.stat.auckland.ac.nz/~wild/09.USCOTSTalk.html
- Wild, C.J., M. Pfannkuch, M., Regan, M. and Horton, N.J. (2011). Towards more accessible conceptions of statistical Inference (with Discussion). *Journal of the Royal Statistical Society A*, **174**, 247–295.

# Buyer Beware

want in 20 mins to cover the flavour of a broad sweep

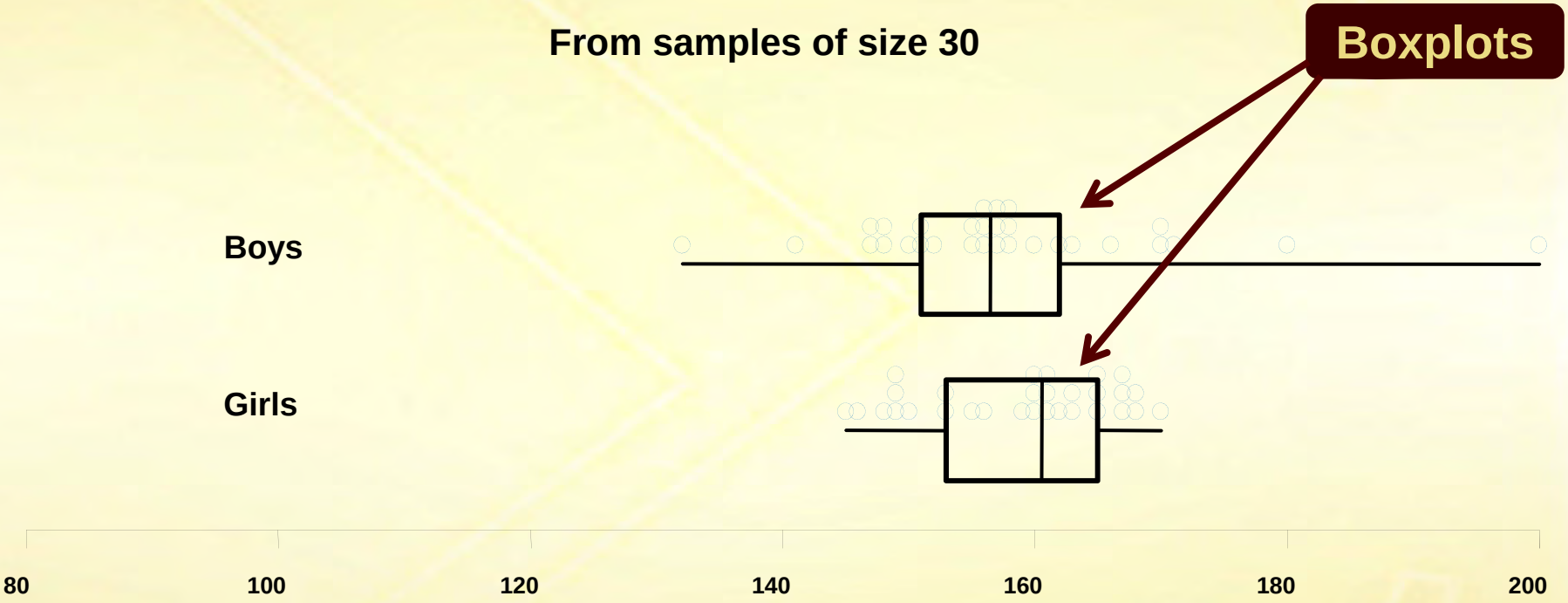
- Corollary: Can't afford to …
  - dot any *i*'s
  - cross any *t*'s
  - qualify any sweeping assertions

- Many of these slides are just springboards for animations

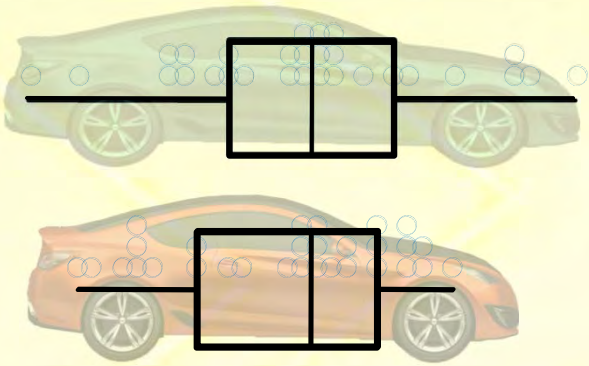
# “Reminders”

**My story needs familiarity with certain forms of animated graphics**

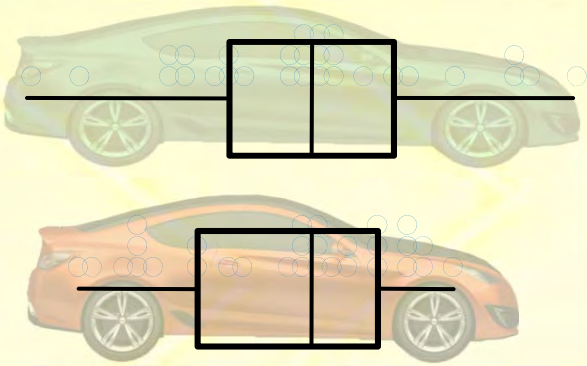
# Comparing heights of boys and girls at age 12



# Learning to see shifts

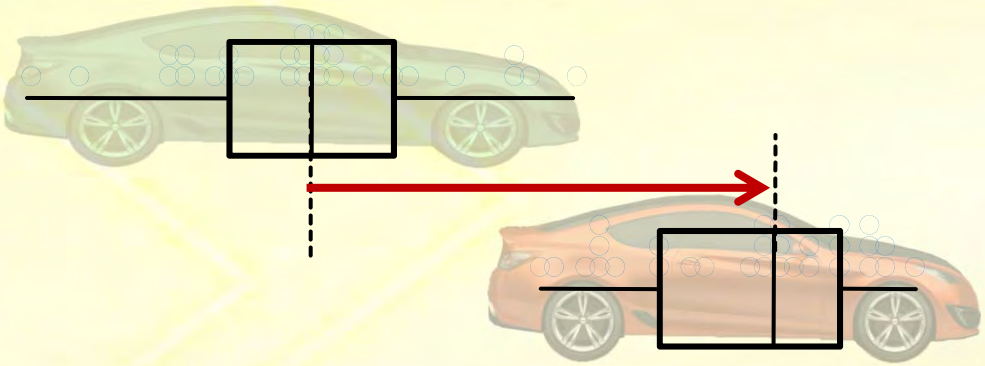


# Learning to see shifts



# Learning to see shifts

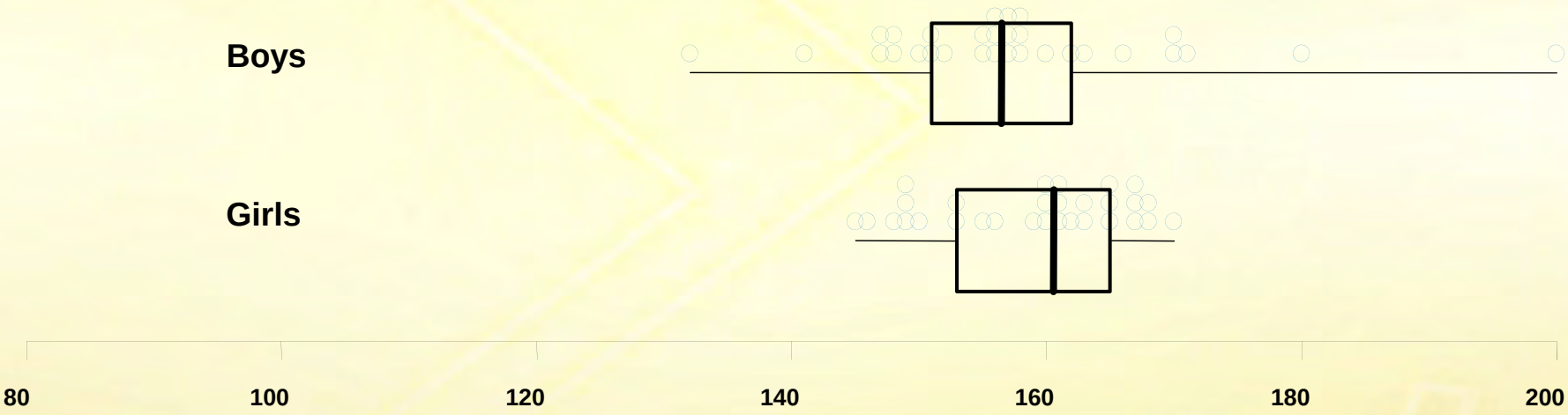
*Obvious measure of **extent of shift ...***



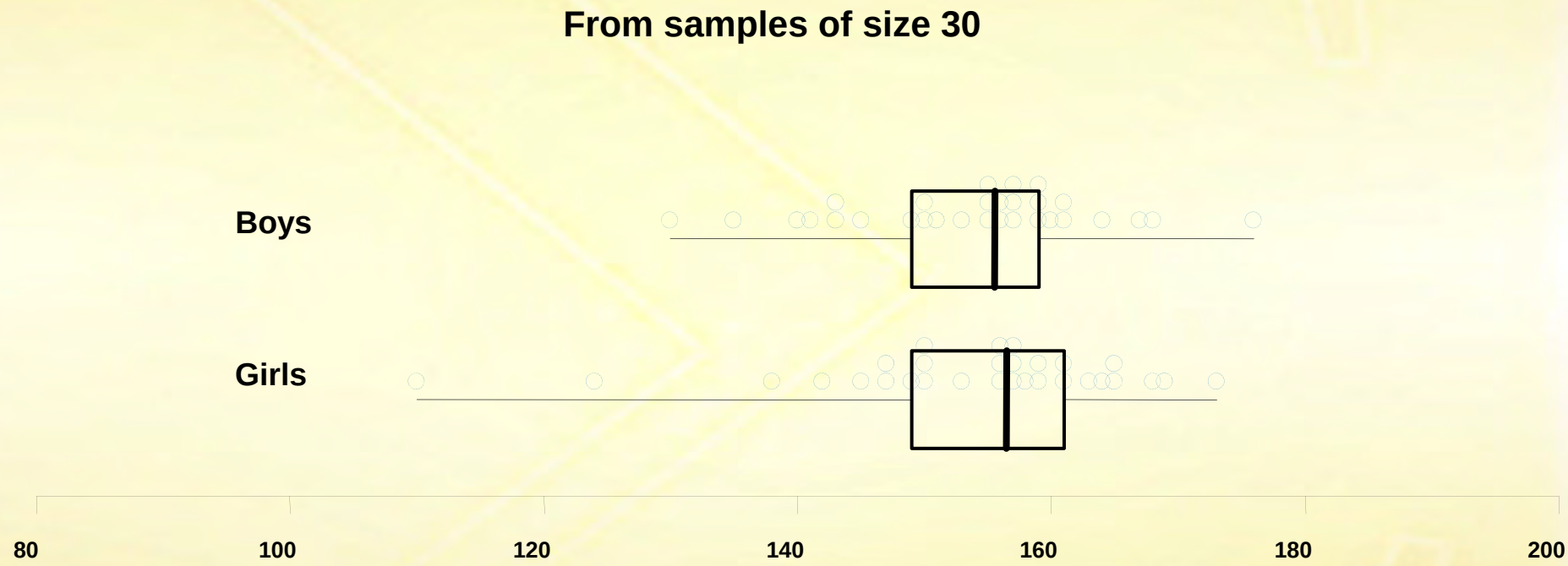
*is **distance between centres***

# Comparing heights of boys and girls at age 12

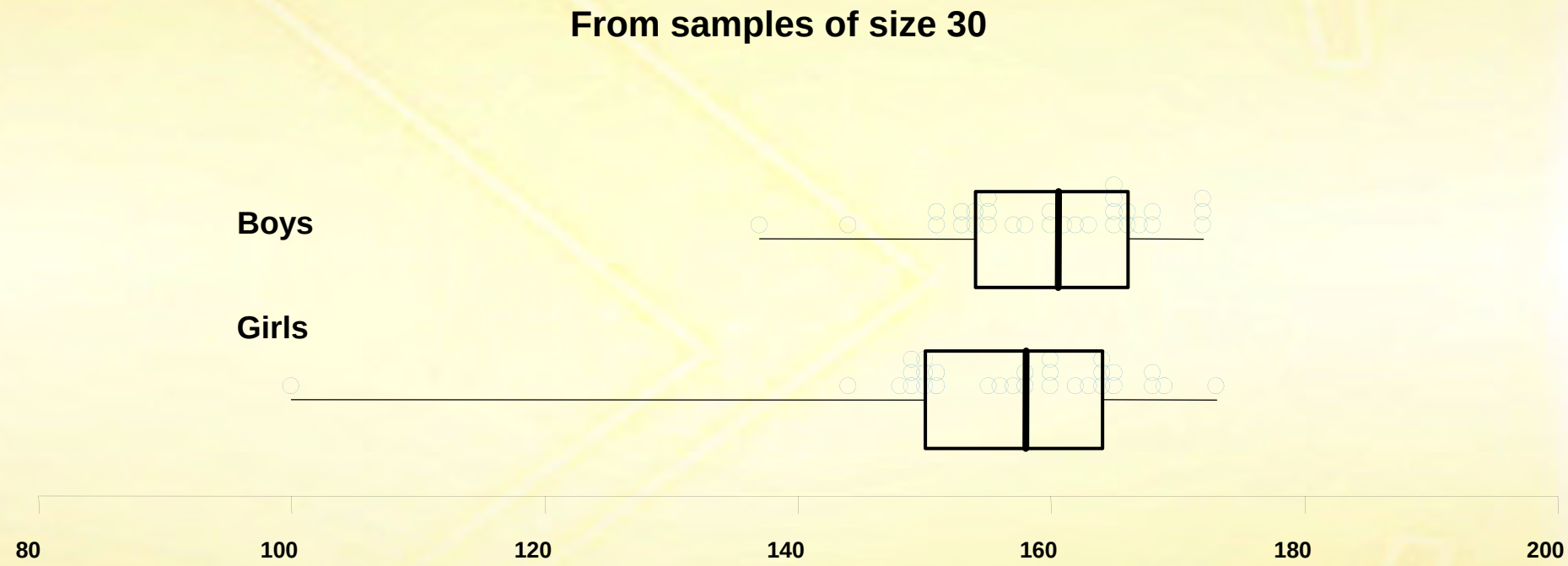
From samples of size 30



# Comparing heights of boys and girls at age 12

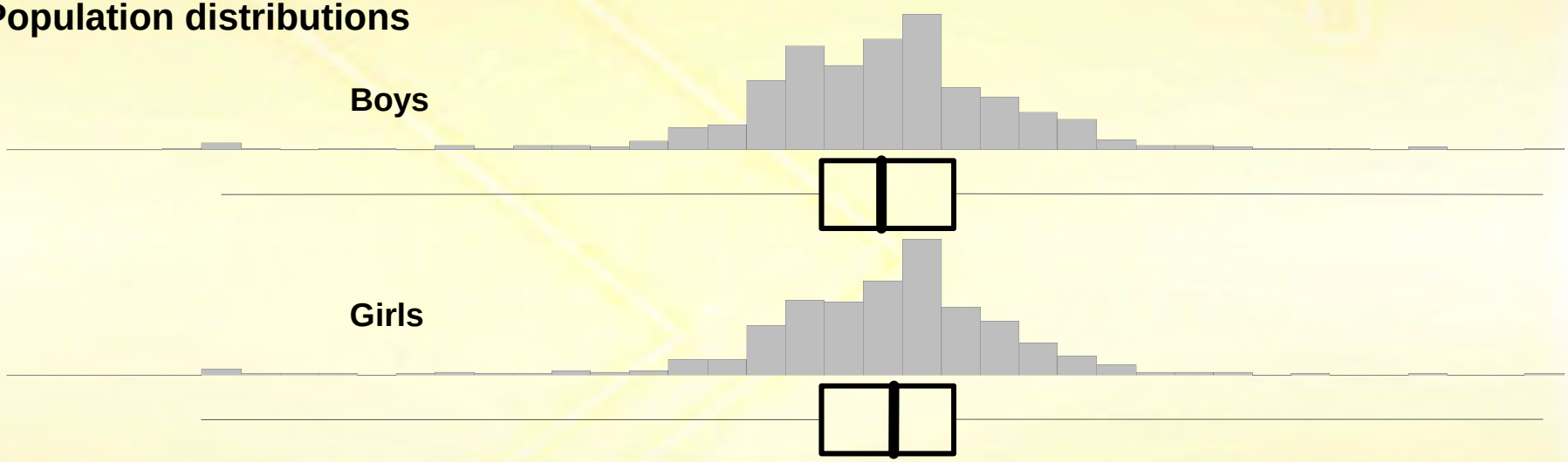


# Comparing heights of boys and girls at age 12



# Comparing heights of boys and girls at age 12

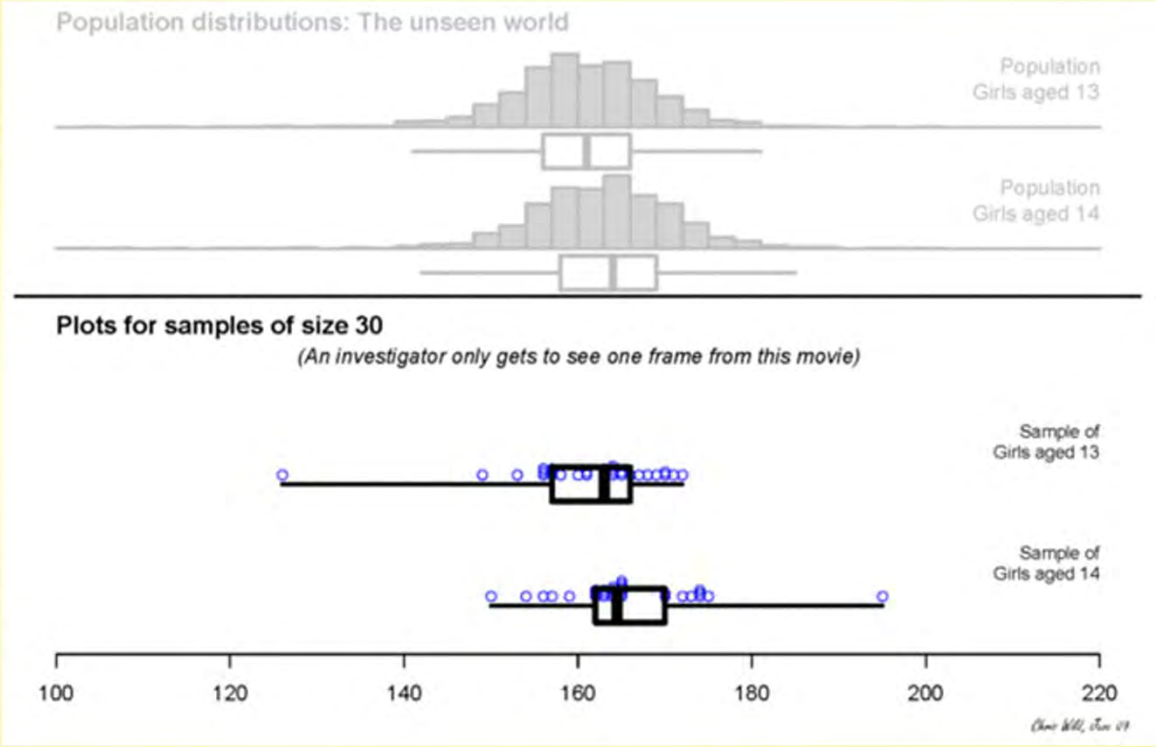
**Population distributions**



# Looking at the world using data is like looking through a window with ripples in the glass

*"What I see …*
*is not quite the way it really is"*

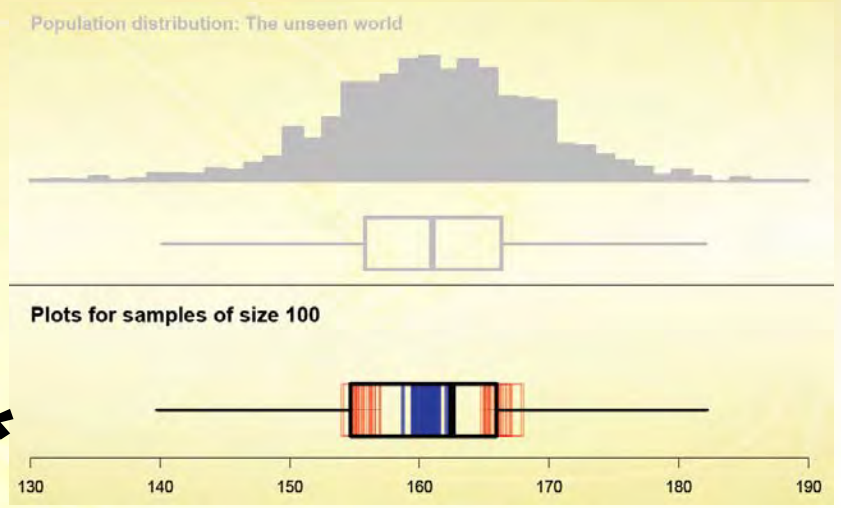
# Dot and Boxplot Animations

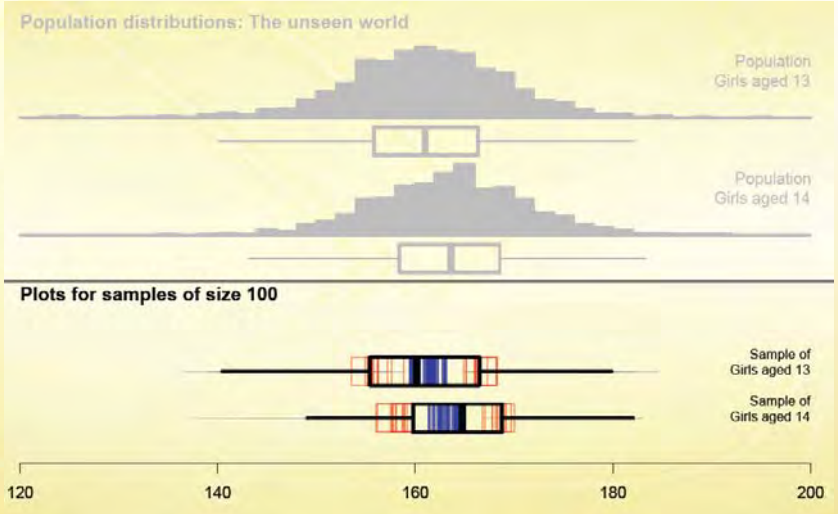


# Boxplots with a Memory

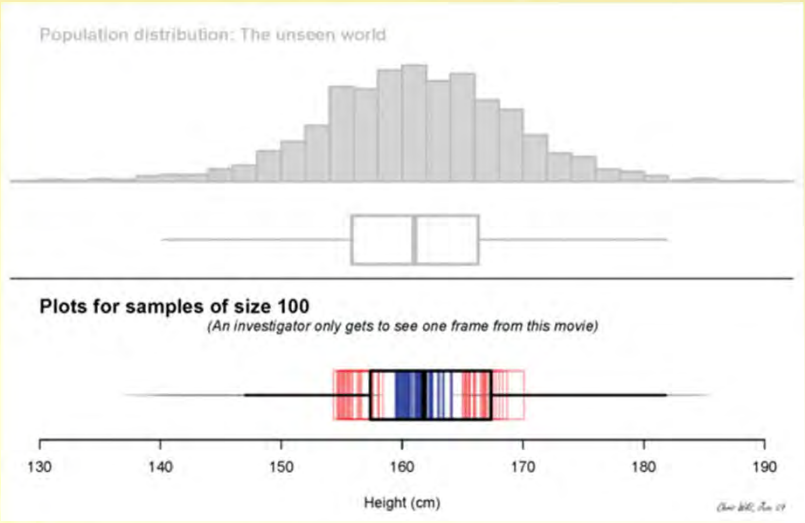
Play

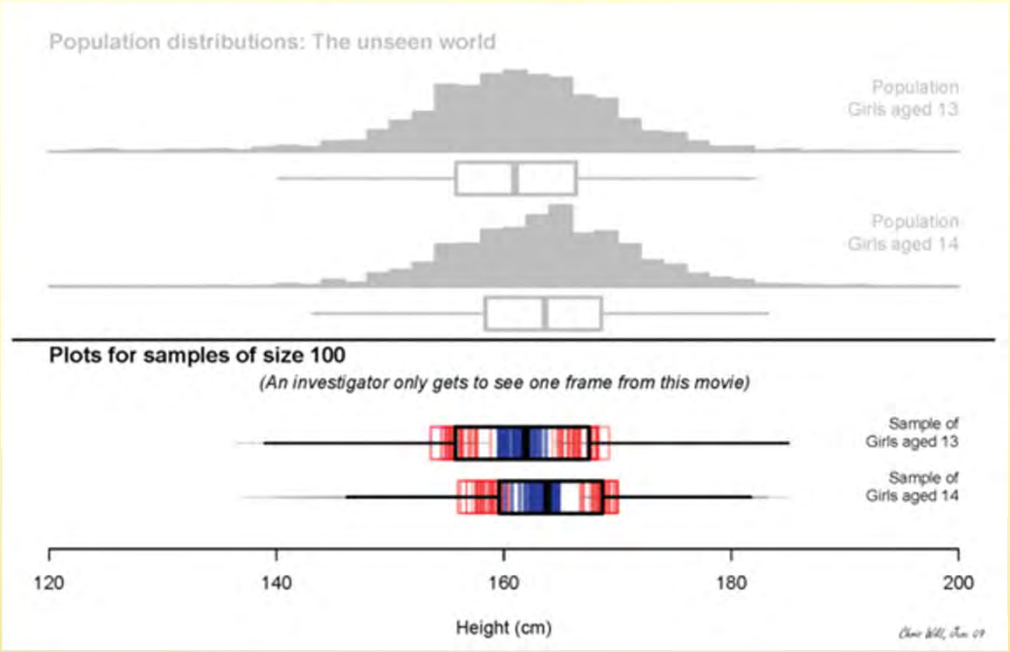
- 1-sample build-up, n=30**



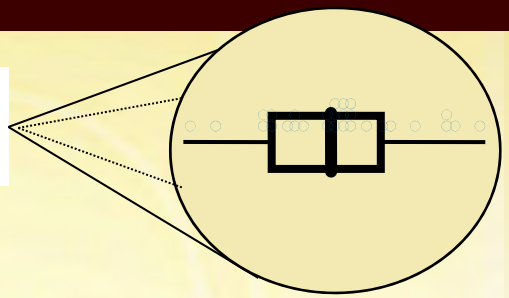
# Boxplots with a Memory



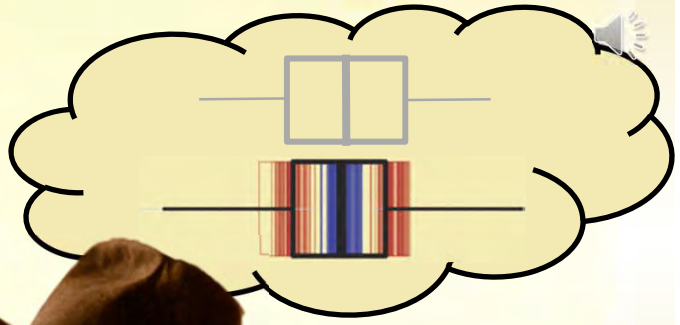




# A new reflex ...

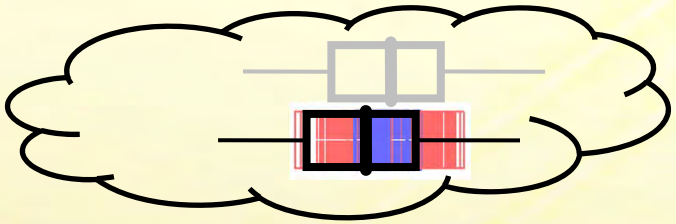
**"Whenever I see ...**



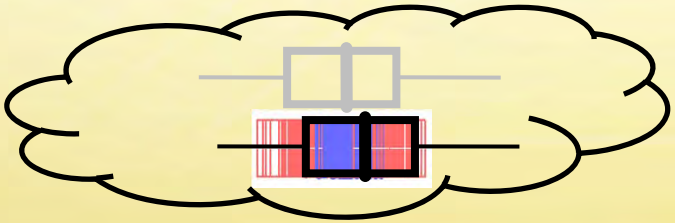
**"I should remember ...**



**"Mine could be like this ..."**

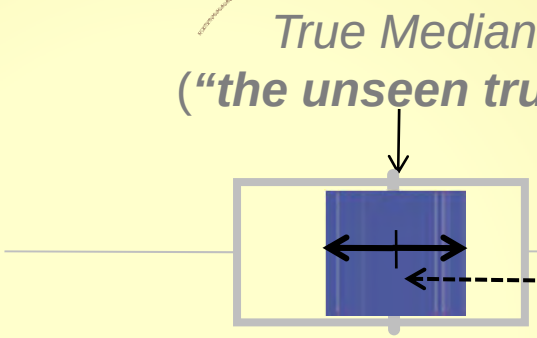


**"Or even this ..."**



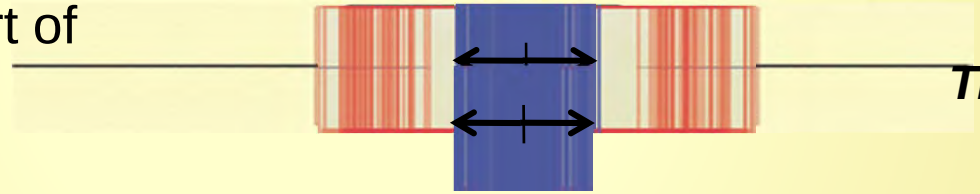
**"I must take this uncertainty**
*about where it really should be*
**into account** (e.g. when I make comparisons)**"**

# Engaging some intuition



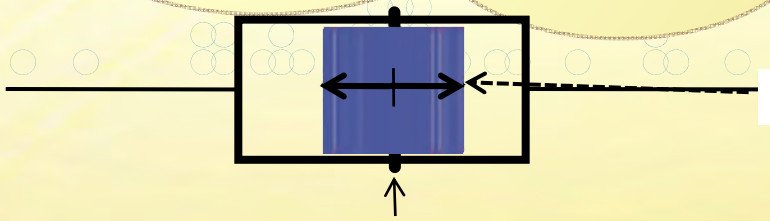

**Data median** *is seldom further* **to try to catch this**

I know this sort of thing happens



***Truth*** *is seldom further from* ***data median*** *than this*

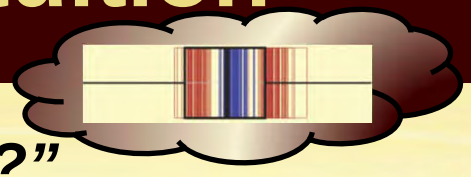
***I*** got this




**Use this …**

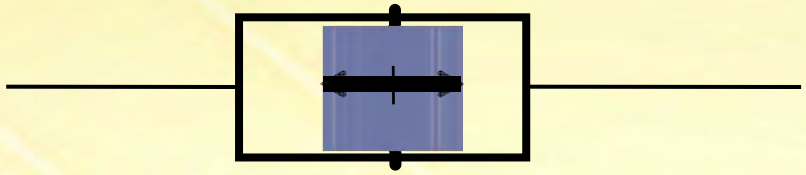
***"but where is truth likely to lie?"***

# Intuition



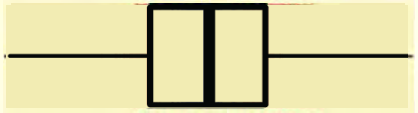
***"Where is truth likely to lie?"***

**I** got this



***Truth** is seldom further from **my data median** than this*

**Problem:** ***I don't actually see width of this "uncertainty" band***



**Why?:** I only see **<u>one</u>** frame of sampling variation movie

**So:** We need some sort of estimate of the width of uncertainty band ***from the single sample itself***

***How ????*** ***Seems impossible!!***

# Enter Brad Efron & the simple Bootstrap

Efron (1979)

*(1996 photo)*

**I wonder** if
"*sampling with replacement from the sample*"
will **mimic** the process of
"*sampling from the population*"

# Bootstrap intervals

**"Re-sampling without replacement"**

- ***What is it?***

# Bootstrap intervals

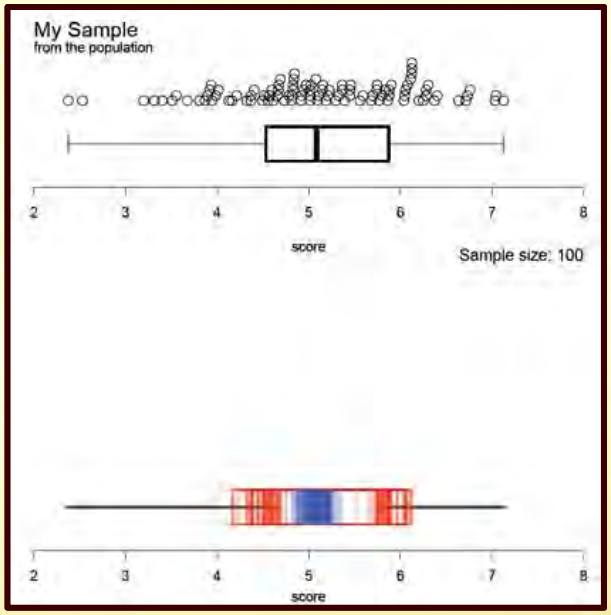
**"Re-sampling without replacement"**

- ***What does it do?***
- ***Why might it work?***

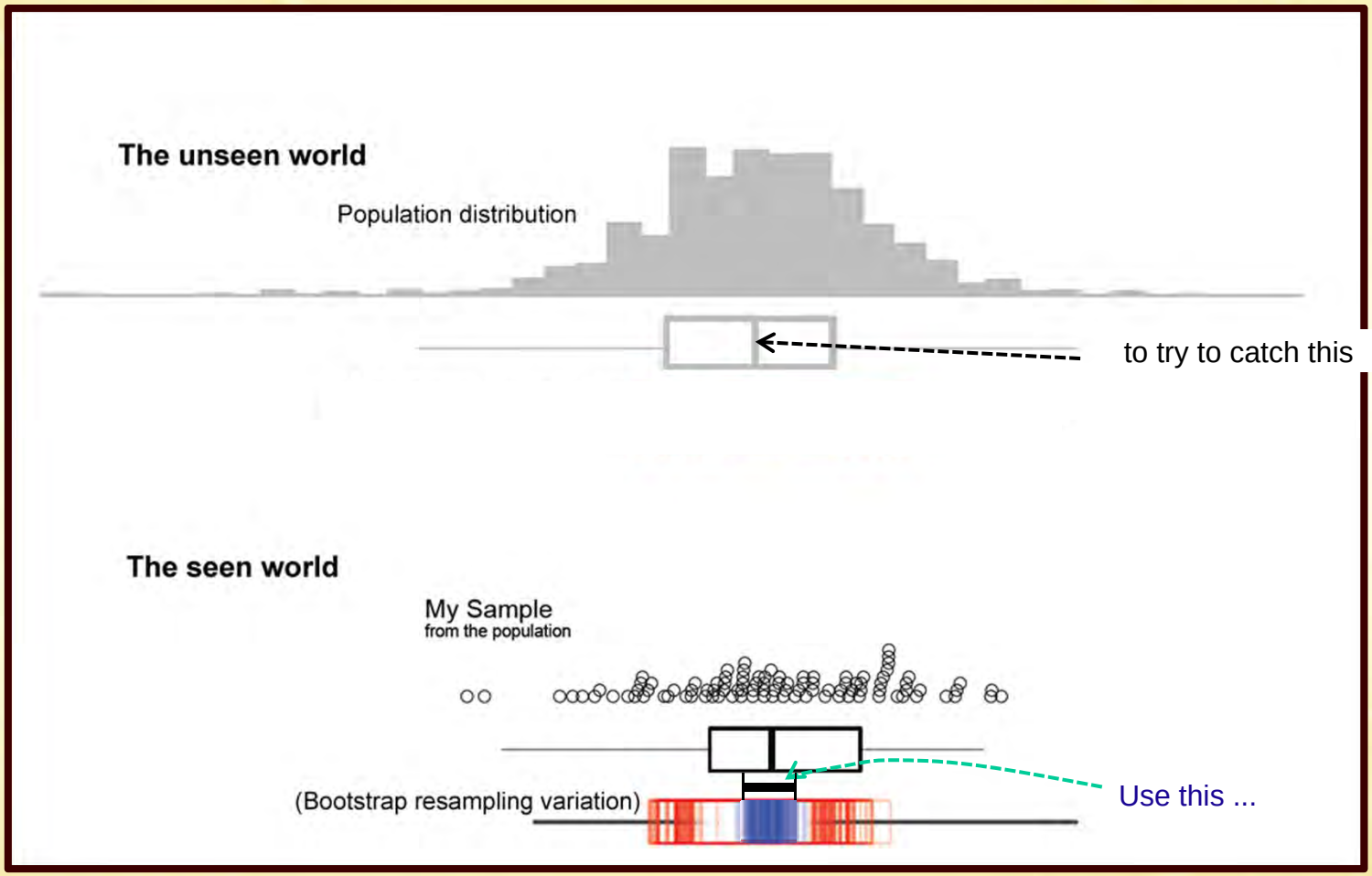
**Plausibility**

# What does bootstrap resampling look like?


**How can we use it?**



# What does bootstrap resampling look like?



# Bootstrap intervals

## Using re-sampling to construct an interval

Construction

- *How is it done?*

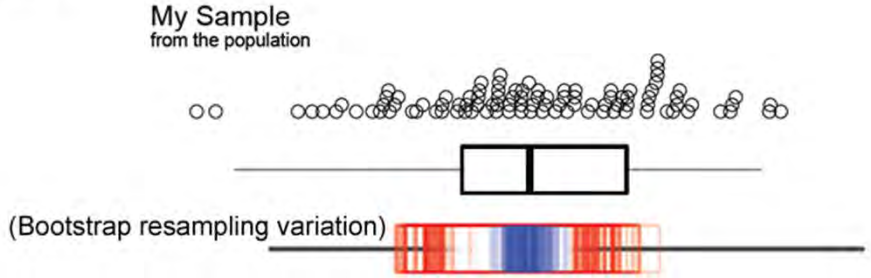
# Bootstrap intervals


Looks plausible …

Does it work ??

"Simulate & see"

The seen world



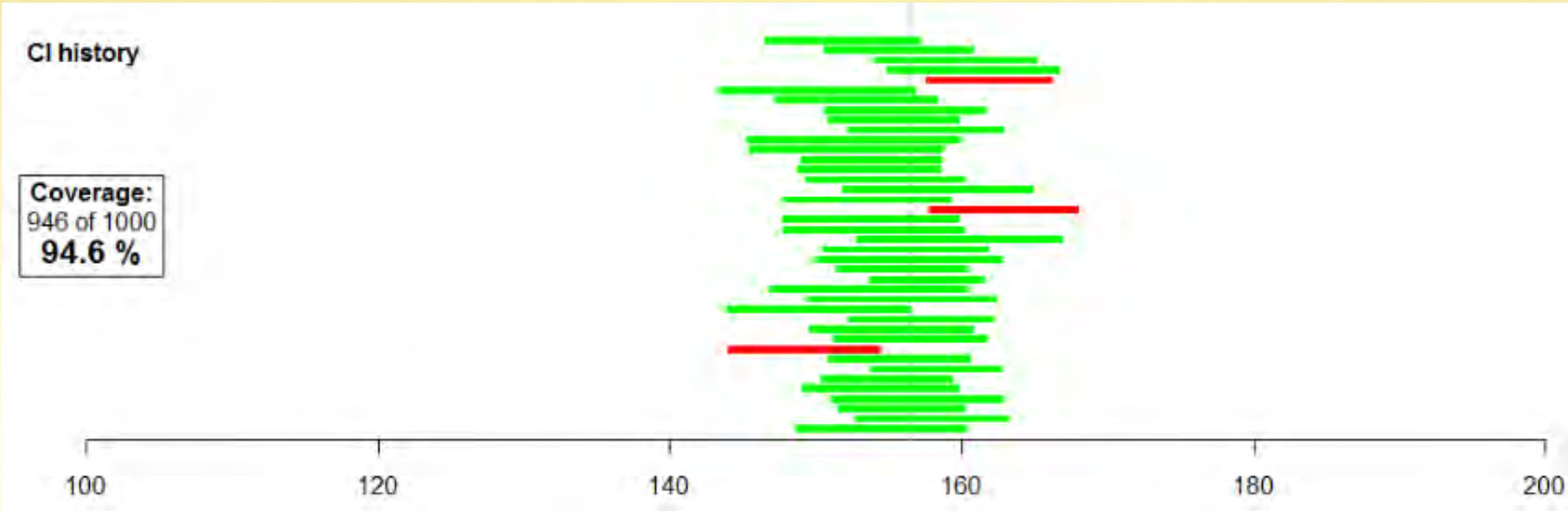
# Bootstrap intervals

## Using re-sampling to construct an interval

Confirmation

***Does this method work?***

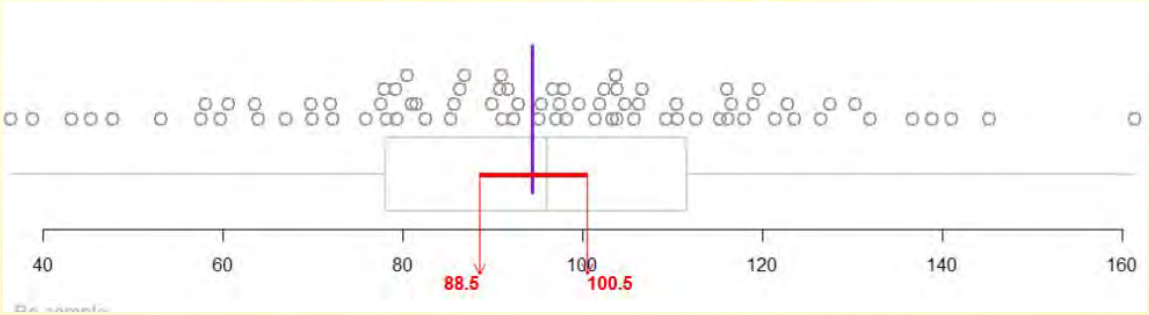
# Lot's of programs that do this




- What we are adding
  - reducing abstraction by making the foundational connections along the way
    - as ***concretely*** as we can

# Bootstrap intervals

## Using re-sampling to construct an interval




Operation

***How do I use it?***

# Bootstrap intervals

## 2-sample shift in centres

# Randomised experiments and Randomisation tests

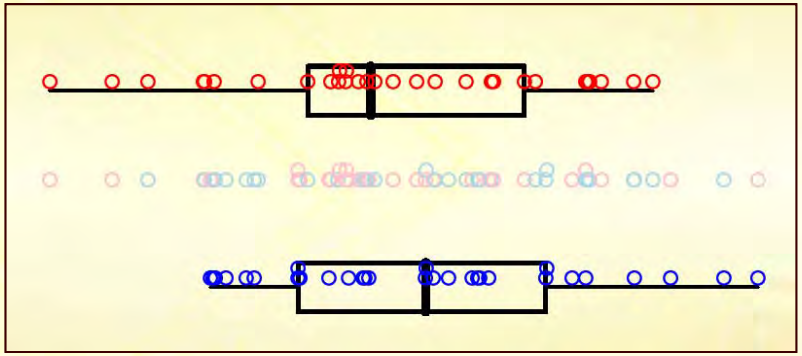
# Experiential Context

- Simple randomised experiment
  - (e.g. drug vs control)
  - Follows experiences on why do randomised experiments
    - Following up message that randomisation is best way we know of balancing groups
      - on unforeseen factors so that apart from treatment we are ***comparing like with like***

# Experiential Context

1st new message:

- ***Randomisation is best, but it is not perfect …***



**"*What I see isn't exactly the way it is*"**

**Why?: *Randomisation variation***

*Randomisation alone can make it look like there is a difference between groups*
*(the apparent differences result simply from who, by chance, ends up in what group)*

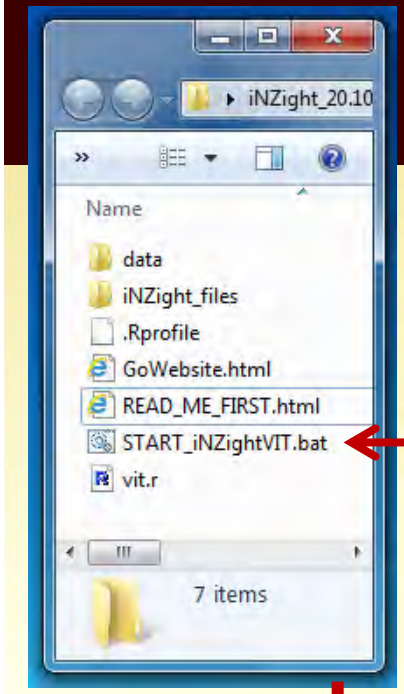
# 2-sample randomisation test visualisation

**Can randomisation alone ("*chance alone*") explain what I'm seeing?**

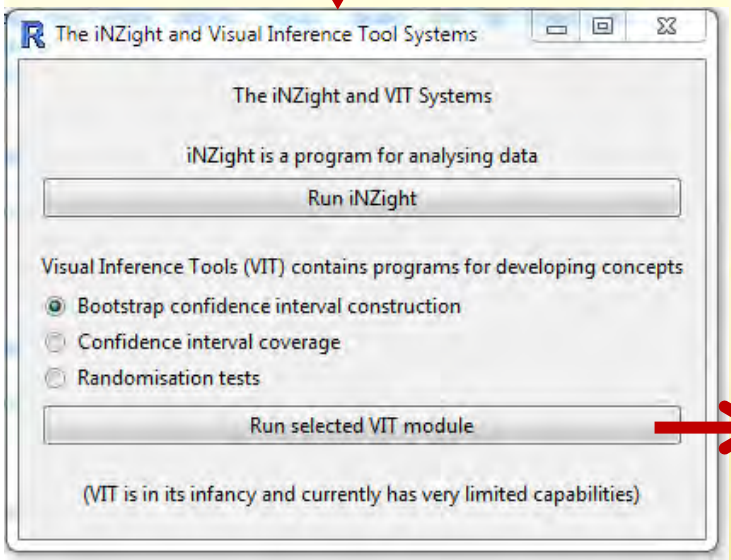
******PUT THIS PHRASE IN THE MOVIE ******

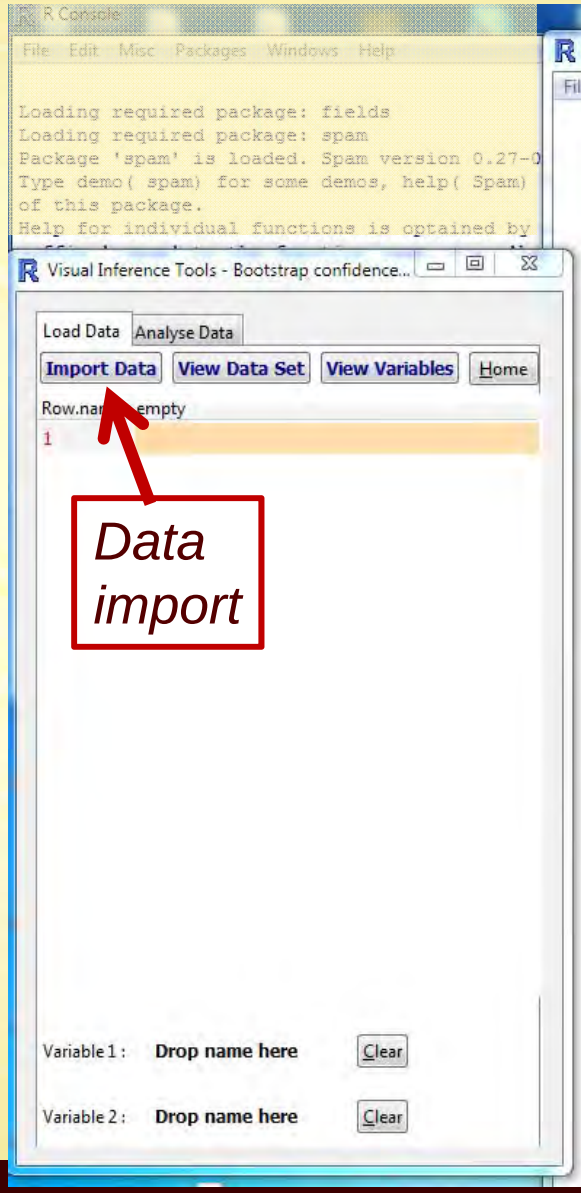
# VIT: Visual Inference Tools

## http://www.stat.auckland.ac.nz/~wild/VIT

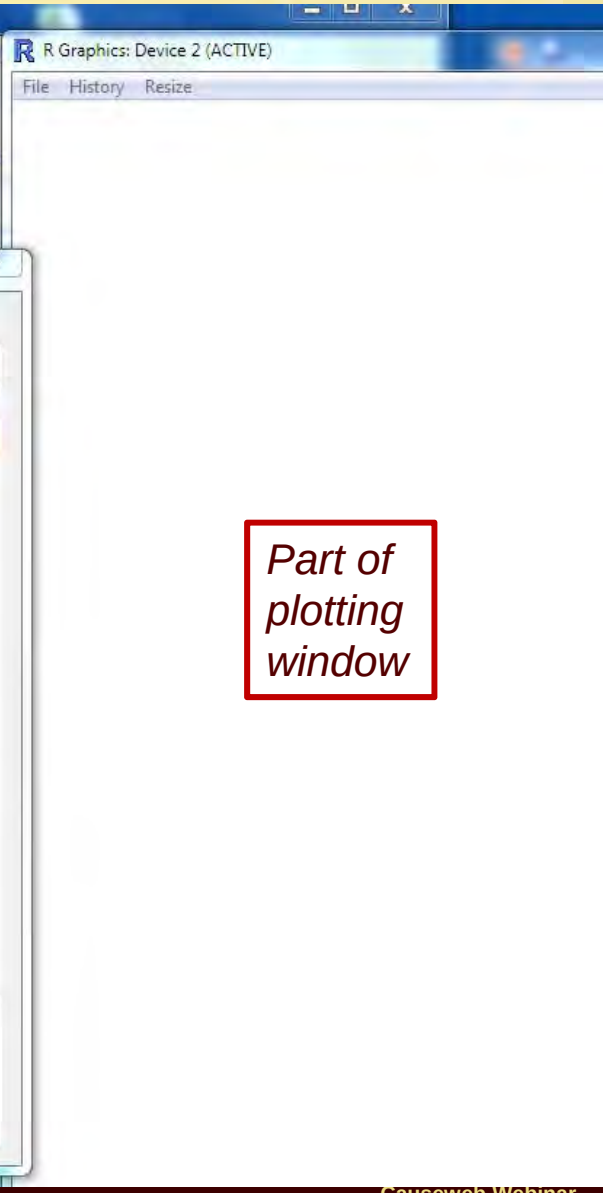



*Double click*

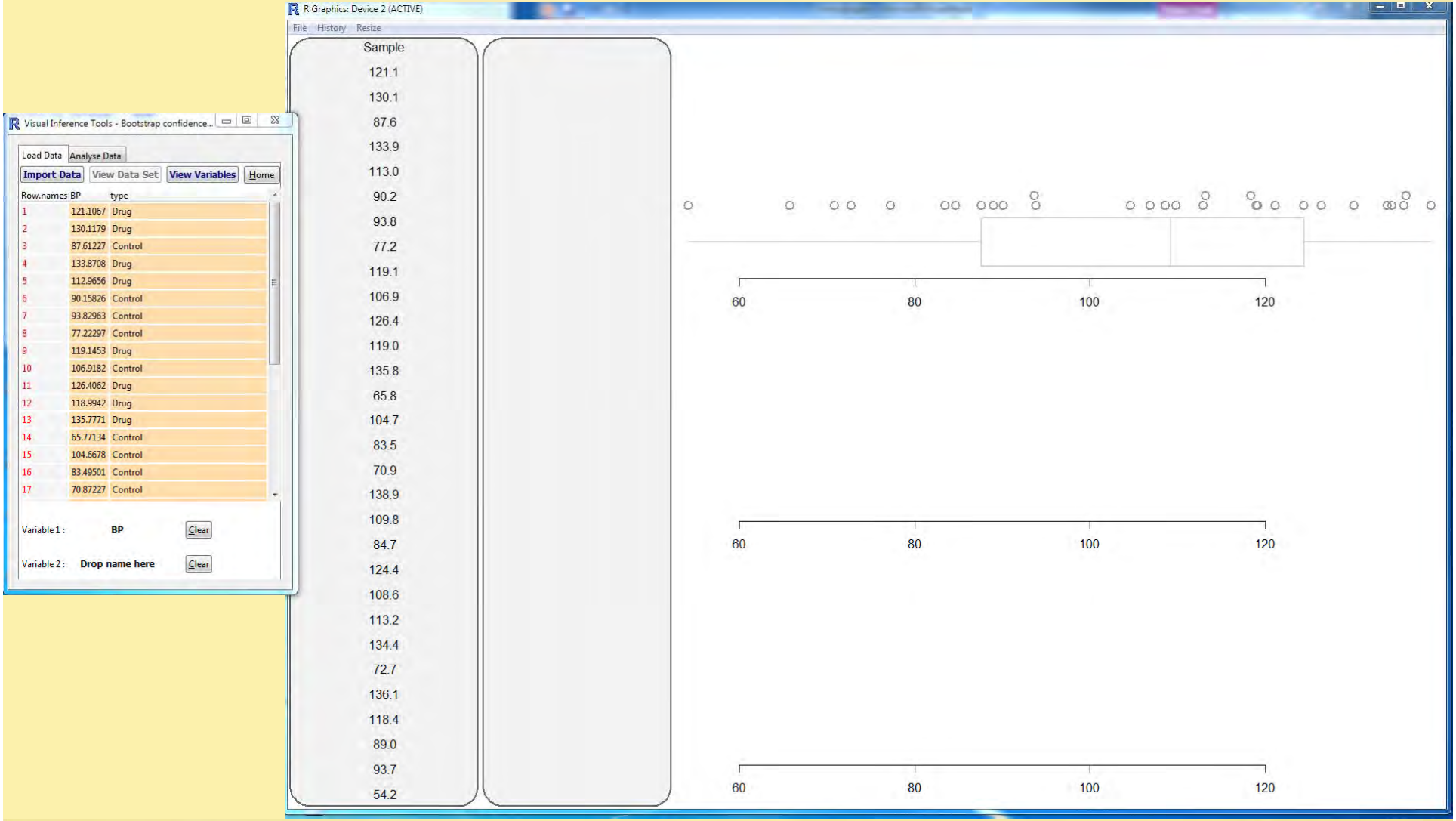



*Data import*




*Part of plotting window*


R Graphics: Device 2 (ACTIVE)
File History Resize
Sample
121.1
130.1
87.6
133.9
113.0
90.2
93.8
77.2
119.1
106.9
126.4
119.0
135.8
65.8
104.7
83.5
70.9
138.9
109.8
84.7
124.4
108.6
113.2
134.4
72.7
136.1
118.4
89.0
93.7
54.2
60
80
100
120
60
80
100
120
60
80
100
120
Visual Inference Tools - Bootstrap confidence...
Load Data
Analyse Data
Import Data
View Data Set
View Variables
Home
Row.names BP type
1 121.1067 Drug
2 130.1179 Drug
3 87.61227 Control
4 133.8708 Drug
5 112.9656 Drug
6 90.15826 Control
7 93.82963 Control
8 77.22297 Control
9 119.1453 Drug
10 106.9182 Control
11 126.4062 Drug
12 118.9942 Drug
13 135.7771 Drug
14 65.77134 Control
15 104.6678 Control
16 83.49501 Control
17 70.87227 Control
Variable 1 : BP
Clear
Variable 2 : Drop name here
Clear

# VIT: In the can or coming sooner or later

http://www.stat.auckland.ac.nz/~wild/VIT

- Bootstrap CIs
  - 1-sample mean/median & proportion (could add LQ, UQ)
  - 2-sample differences in mean/median & proportion
  - Regression slope
  - 1-sample IQR?
- Randomisation tests
  - 2-sample differences in mean/median & proportions
  - *k*-sample differences in mean/median & proportions
- Confidence Interval Coverage
  - 1-sample mean/median & proportion (could add LQ, UQ)

# VIT: Visual Inference Tools

**http://www.stat.auckland.ac.nz/~wild/VIT**

- what currently exists for anyone interested in playing with it
  - Includes download & instructions
- on an as-is, where-is basis - will update as we fine-tune, fix bugs and build additional modules

## Programming Credits

**Garrett Grolemund** *(Rice)*

(New) **Infrastructure**

**Current Team** *(UoA)*

**Ben Stevenson**

**Simon Potter**

**Keng Hao** (Danny) **Chang**

**Jieping** (Vivian) **Li**

**Earlier Prototypes** *(UoA)*

# Some of my related webpages

**VIT: Visual Inference Tools**

- http://www.stat.auckland.ac.nz/~wild/VIT

***iNZight*: A data analysis system with a particularly short learning curve**

- http://www.stat.auckland.ac.nz/~wild/iNZight

**Bootstrap animations**

- http://www.stat.auckland.ac.nz/~wild/BootAnim

**Animations used in USCOTS 2009 and Wild, Pfannkuch, Regan & Horton, 2011, JRSSA**

- http://www.censusatschool.org.nz/2009/informal-inference/WPRH/

**Statistical thinking diagrams and models**

- http://www.stat.auckland.ac.nz/~wild/StatThink

**USCOTS 2009 Keynote Talk "Early Statistical inferences: The Eyes Have It"**

(Movie + Animations + Slides)

- http://www.stat.auckland.ac.nz/~wild/09.USCOTSTalk.html

# Thank you