Foundations in Probability that Support Statistical Reasoning

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CAUSE Webinar - 02/09/10

Literacy, Reasoning, & Thinking

Statistical Literacy
Involves understanding and using the basic language and tools of statistics: knowing what statistical terms mean, understanding the use of statistical symbols, and recognizing and being able to interpret representations of data.

Statistical Reasoning

Statistical Thinking

Definitions used in the ARTIST database to classify three types of assessment tasks https://app.gen.umn.edu/artist/

Literacy, Reasoning, & Thinking

Statistical Literacy

Statistical Reasoning

Is the way people reason with statistical ideas and make sense of statistical information. Statistical reasoning may involve connecting one concept to another (e.g., center and spread) or may combine ideas about data and chance. Reasoning means understanding and being able to explain statistical processes, and being able to fully interpret statistical results.

Statistical Thinking

Definitions used in the ARTIST database to classify
three types of assessment tasks   https://app.gen.umn.edu/artist/

Literacy, Reasoning, & Thinking

Statistical Literacy

Statistical Reasoning

Statistical Thinking
Involves an understanding of why and how statistical investigations are conducted. This includes recognizing and understanding the entire investigative process (from question posing to data collection to choosing analyses to testing assumptions, etc.), understanding how models are used to simulate random phenomena, understanding how data are produced to estimate probabilities, recognizing how, when, and why existing inferential tools can be used, and being able to understand and utilize the context of a problem to plan and evaluate investigations and to draw conclusions.

Definitions used in the ARTIST database to classify three types of assessment tasks https://app.gen.umn.edu/artist/

Foundations in Probability that Support Statistical Reasoning *and Thinking*

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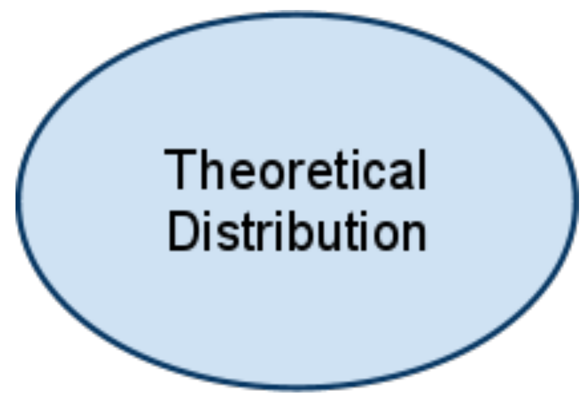
Statistical reasoning may involve connecting one concept to another and may combine ideas about data and chance ...

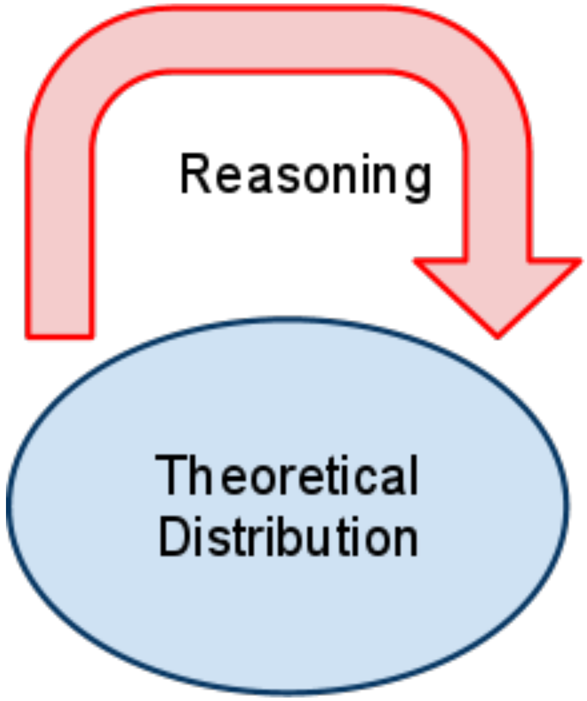
How can we help students connect observations from empirical data (probability in reality) and expected results based on a theoretical model of probability to inform their judgments and inferences?
(e.g., Jones, 2005; Jones et al, 2007; Parzysz, 2003).

Some researchers have begun to tackle how to help students develop notions of informal inference that include examining data sampled (randomly) from finite populations, and data generated from random phenomena that have an underlying probability distribution which may be unknown
(e.g. Pratt, Johnston-Wilder, Ainley, & Mason, 2008; Stohl, & Tarr, 2002; Zieffler, Garfield, delMas, Reading, 2008).

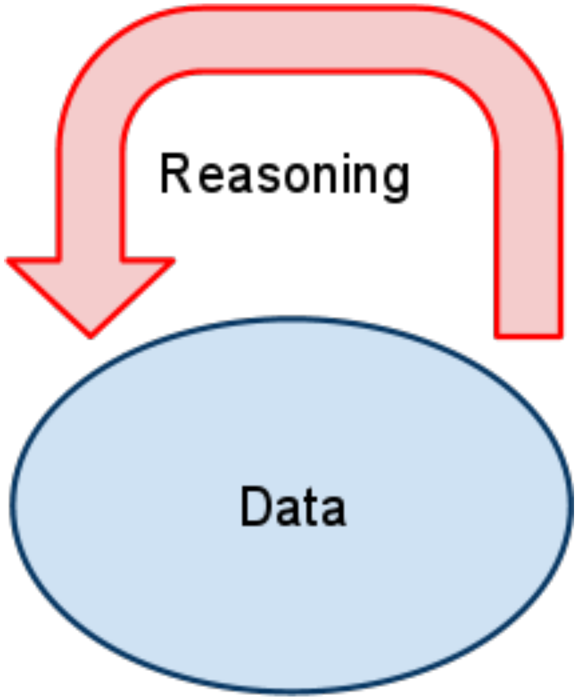
We are not interested in arguing that traditional probability as taught in schools is necessary for statistical reasoning.
However, we do argue that there are various types of probabilistic reasoning that are important to statistical reasoning and thinking.


Theoretical
Distribution

Mathematical Probability



Data Analysis



Probabilistic Phenomenon

Phenomena : an object or aspect known through the senses rather than by thought or intuition

phenomena. (2010). In Merriam-Webster Online Dictionary.
Retrieved from http://www.merriam-webster.com/dictionary/phenomena

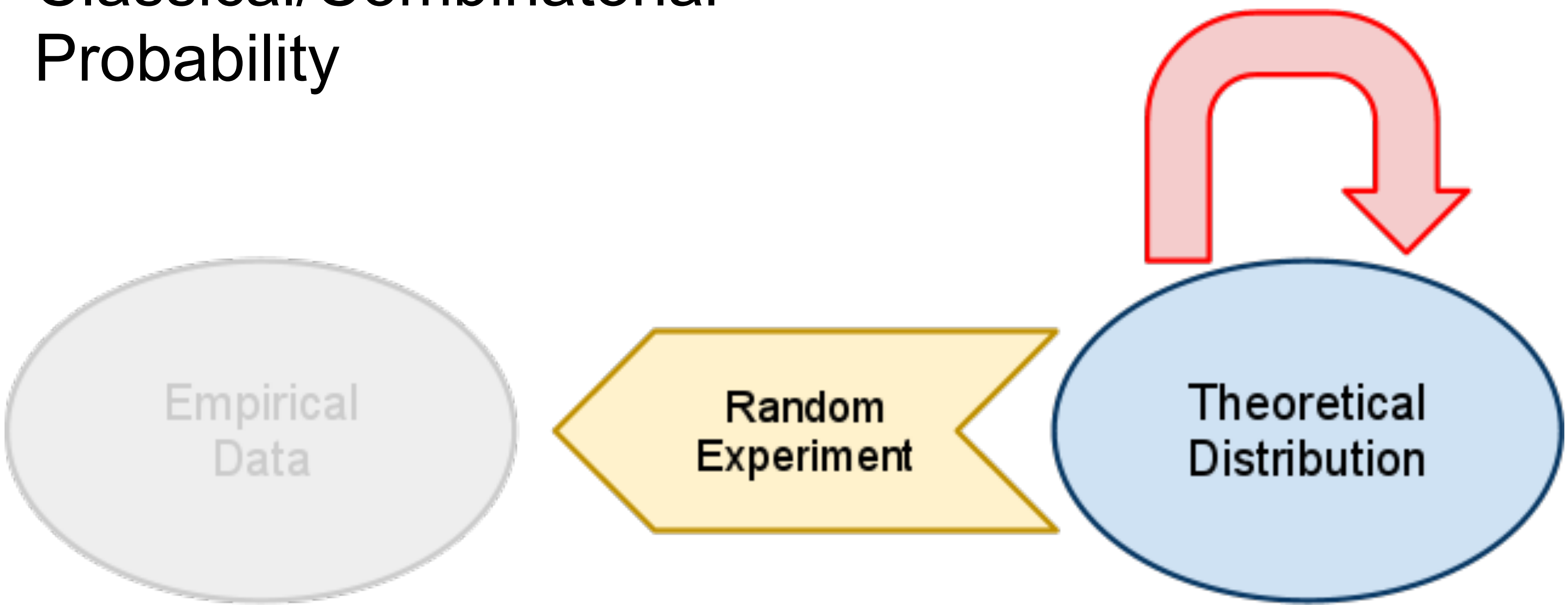
Probabilistic

Phenomena **:** an object or aspect known through the senses rather than by thought or intuition, *upon which a probability structure can be applied*

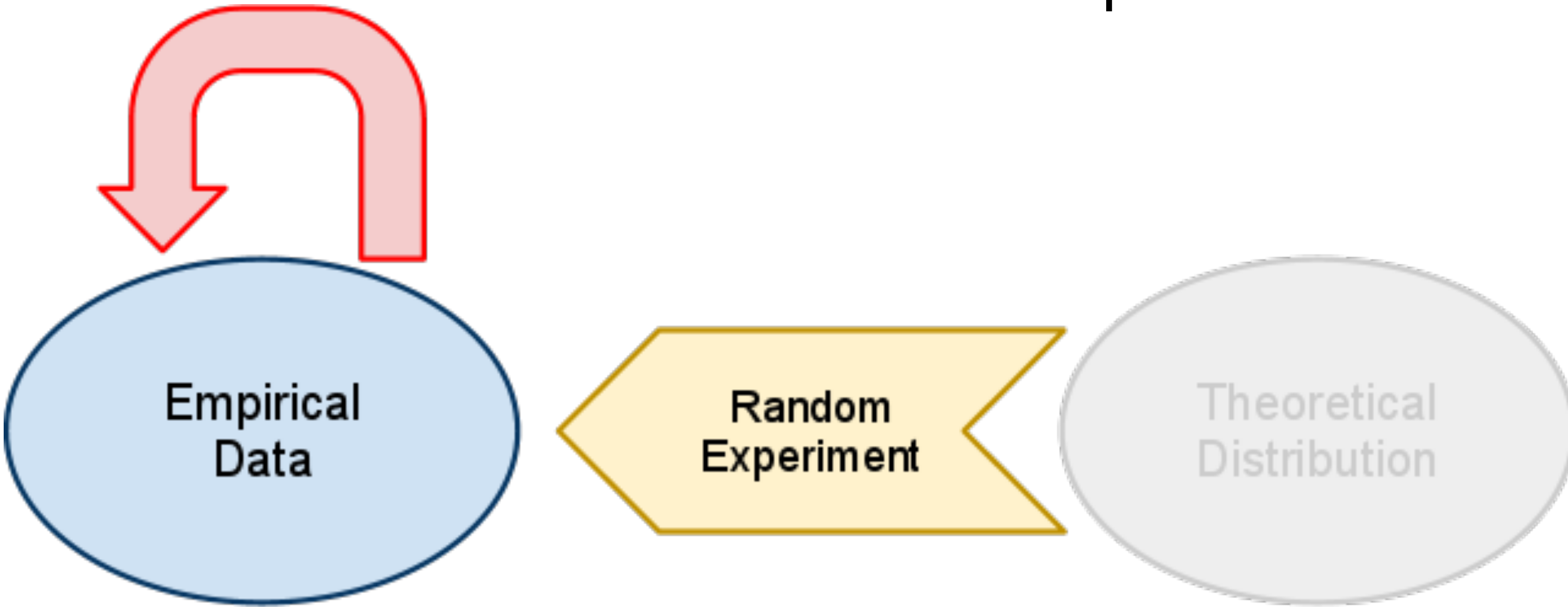
phenomena. (2010). In Merriam-Webster Online Dictionary. Retrieved from http://www.merriam-webster.com/dictionary/phenomena

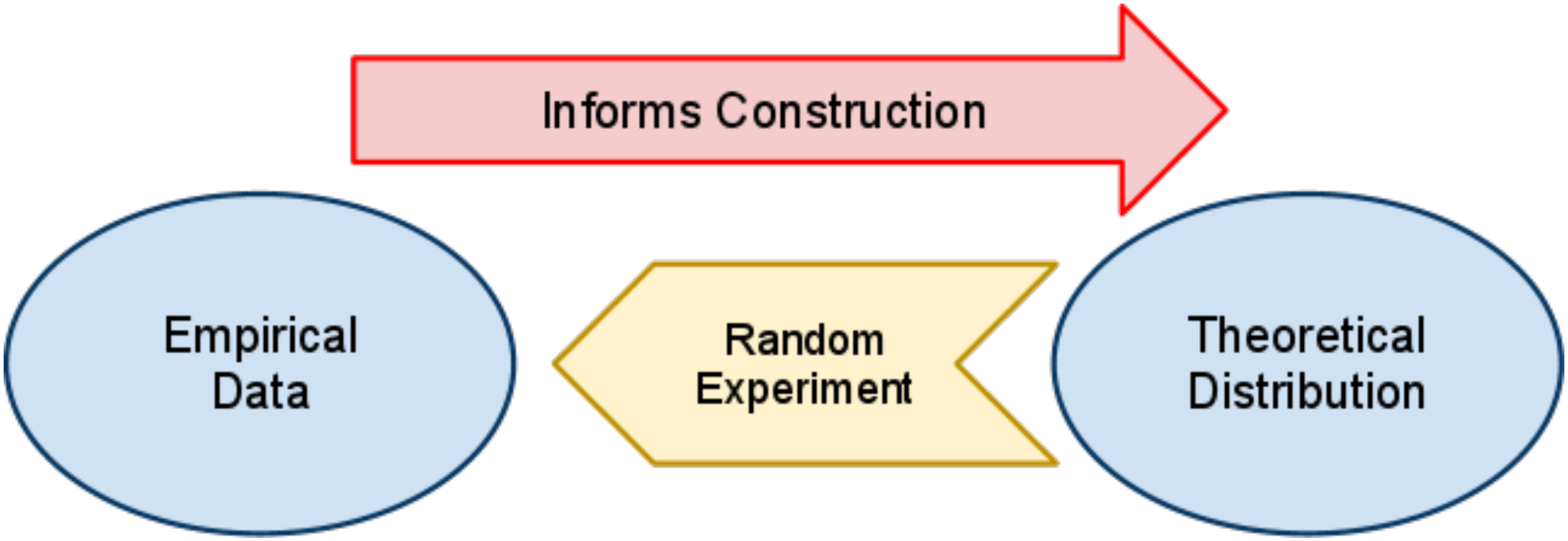
Probabilistic Phenomenon

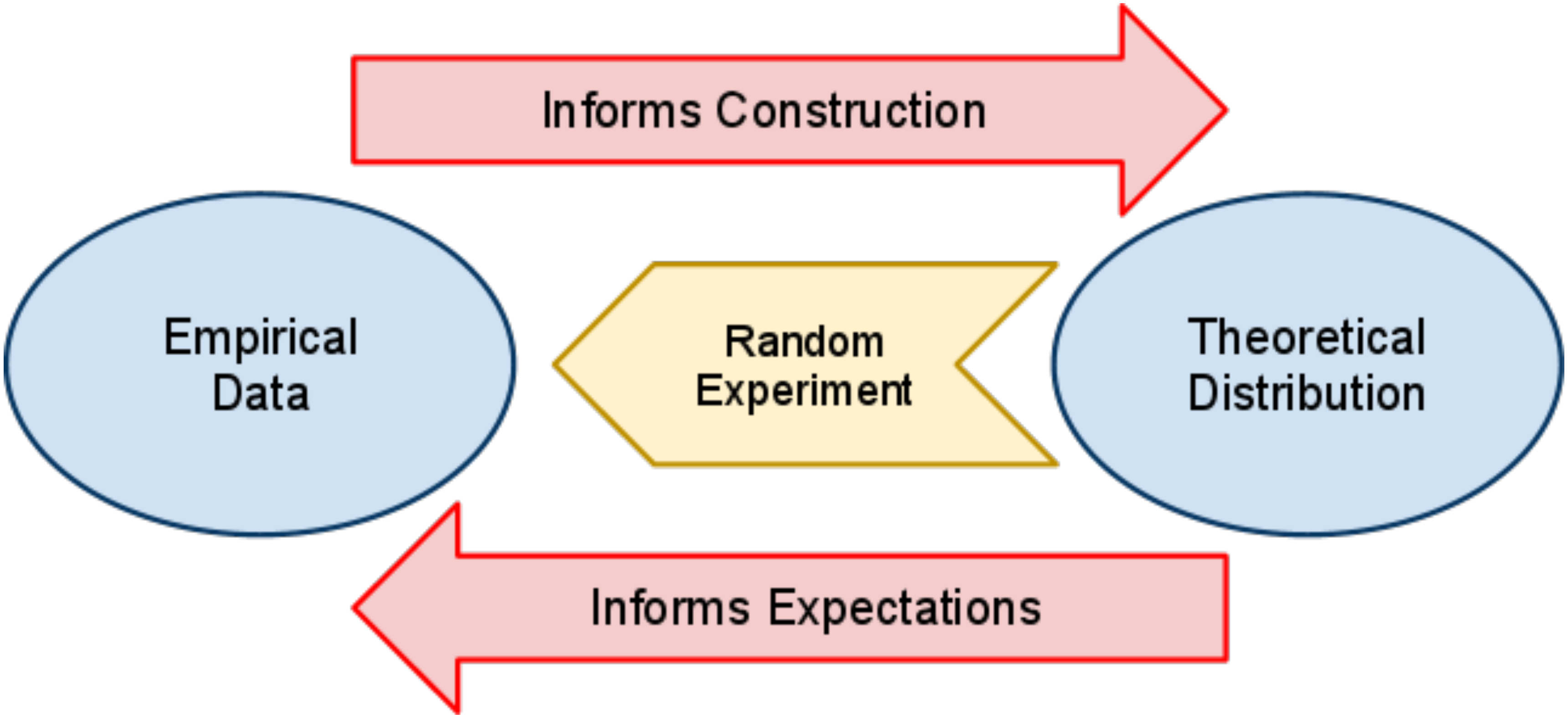
Classical/Combinatorial Probability

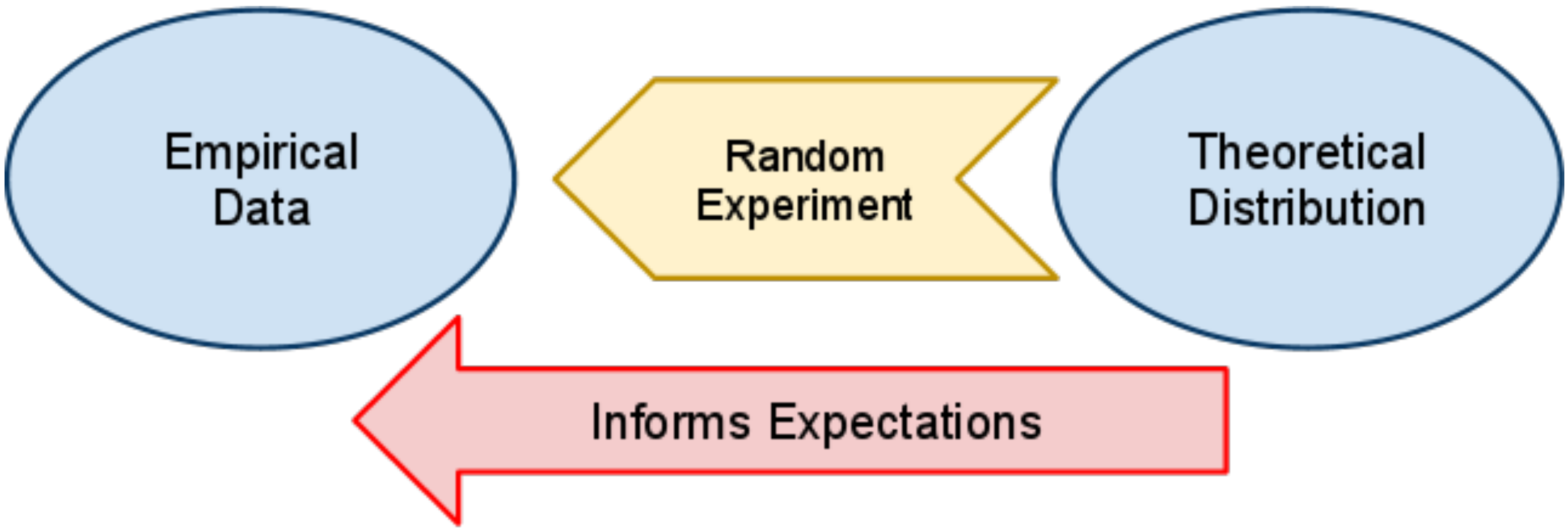


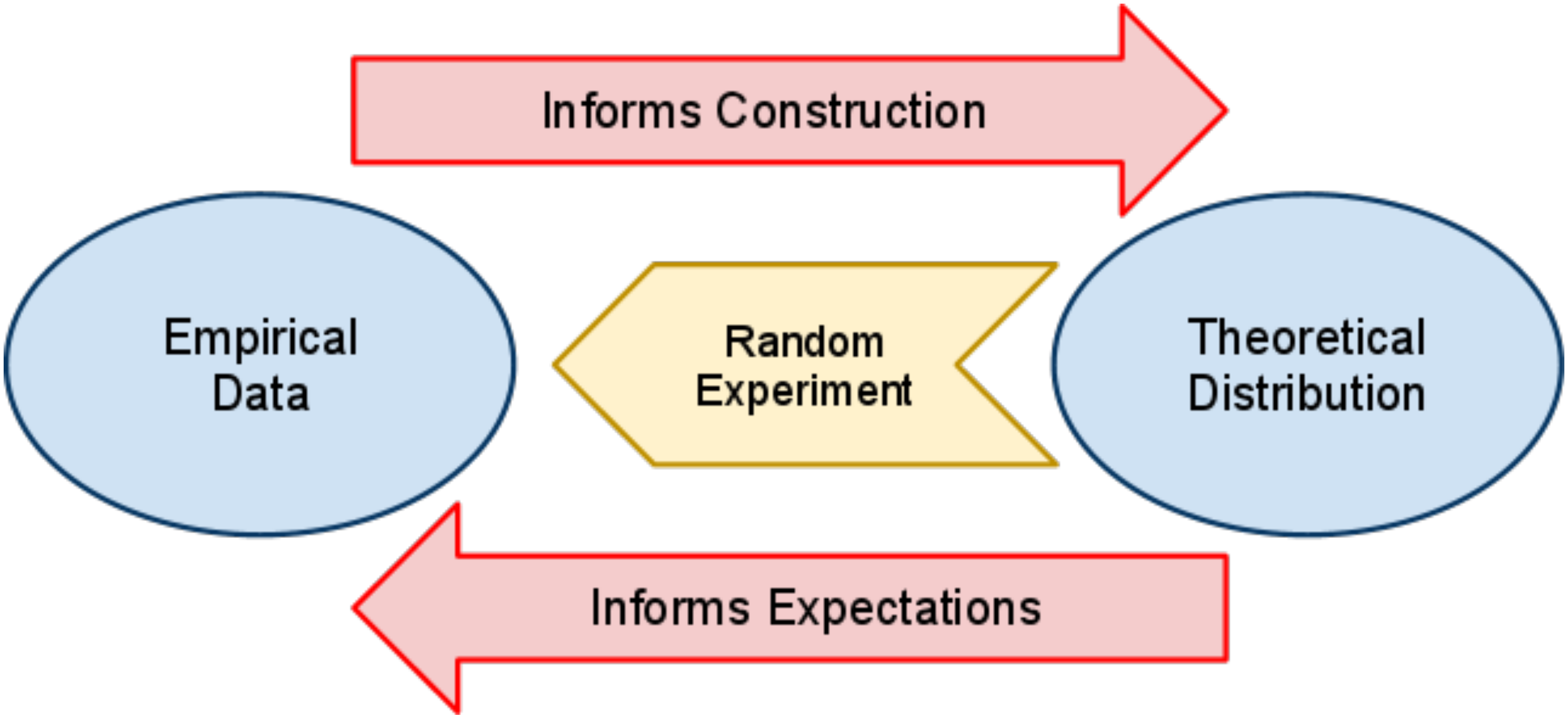
Descriptive Statistics



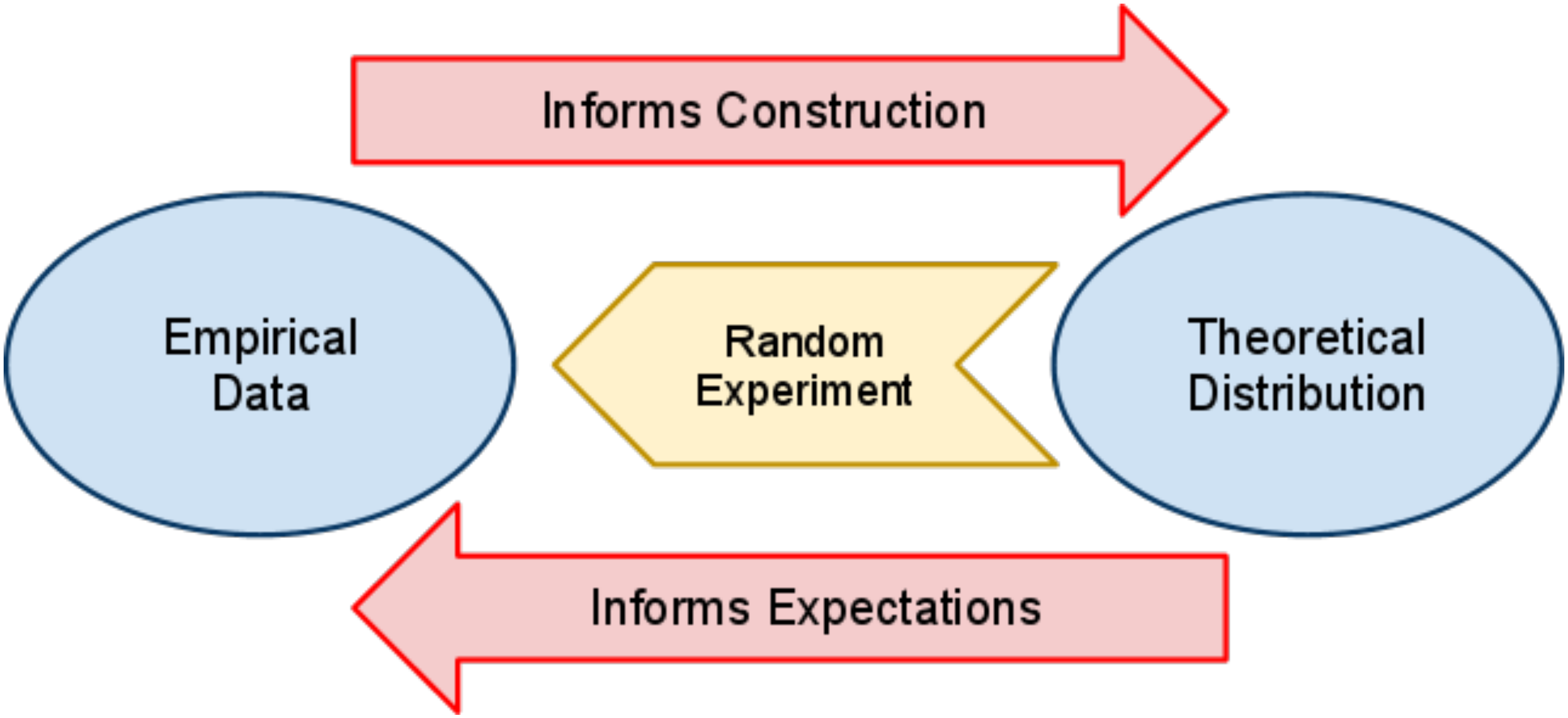

Informs Construction
Empirical Data
Random Experiment
Theoretical Distribution


Informs Construction
Empirical Data
Random Experiment
Theoretical Distribution
Informs Expectations

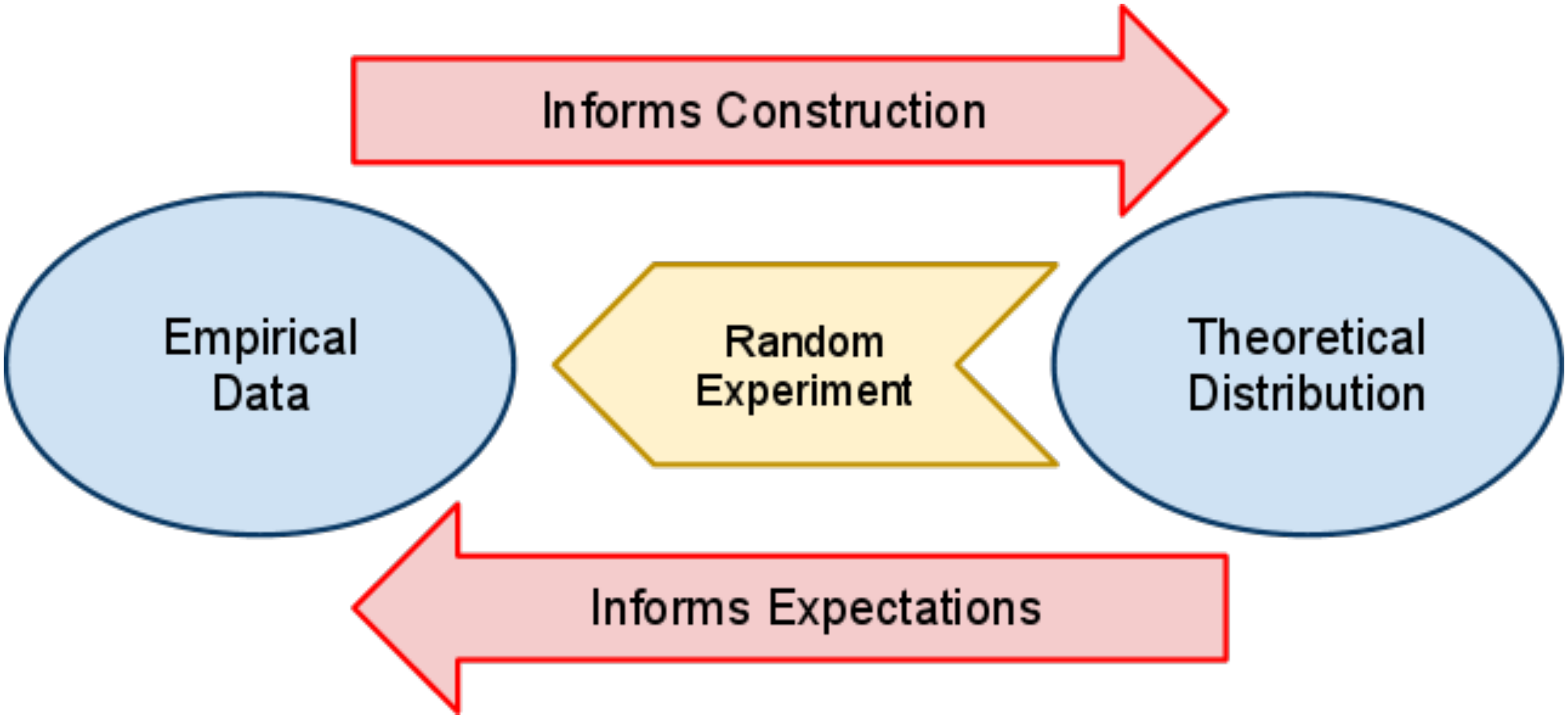

Empirical Data
Random Experiment
Theoretical Distribution
Informs Expectations


Informs Construction
Empirical Data
Random Experiment
Theoretical Distribution
Informs Expectations

Bidirectional Reasoning

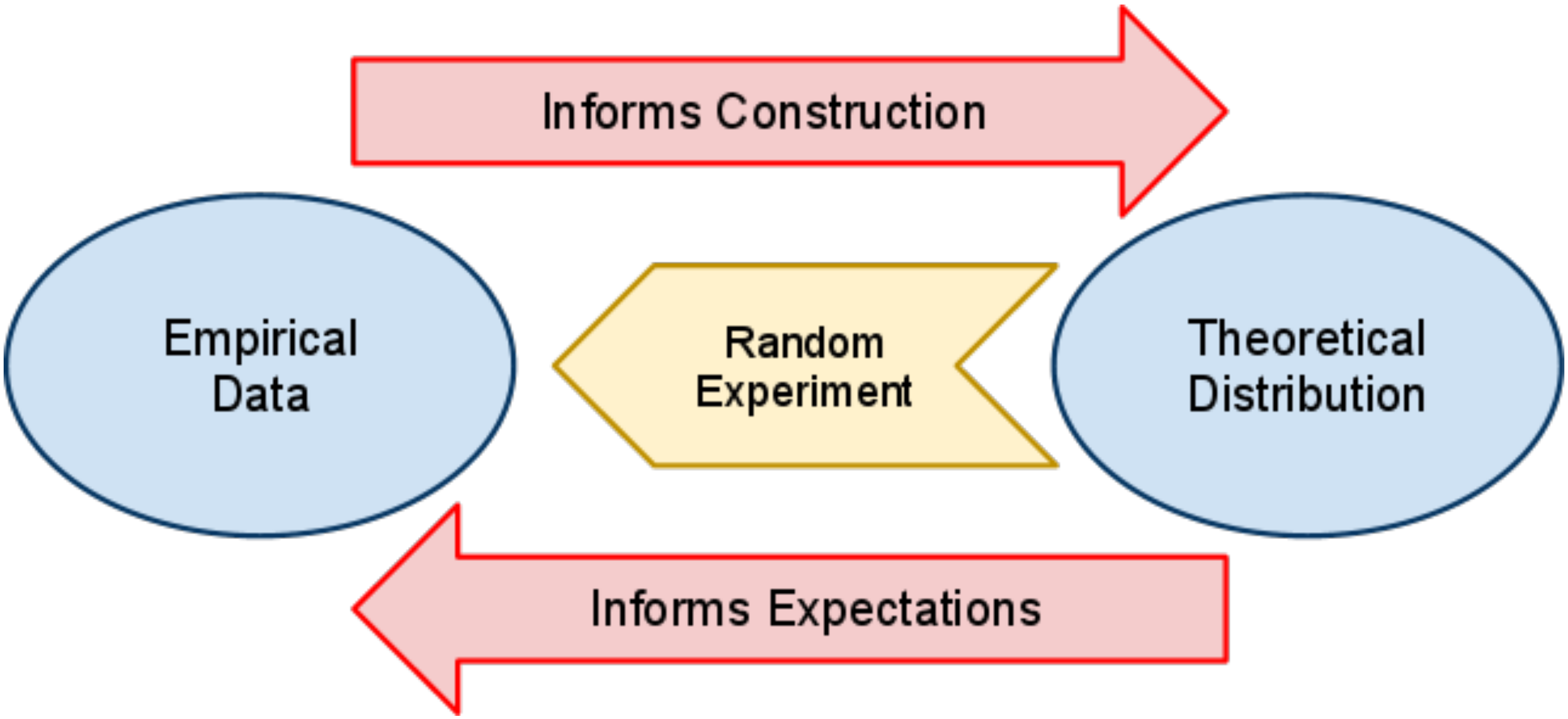


Bidirectional Reasoning



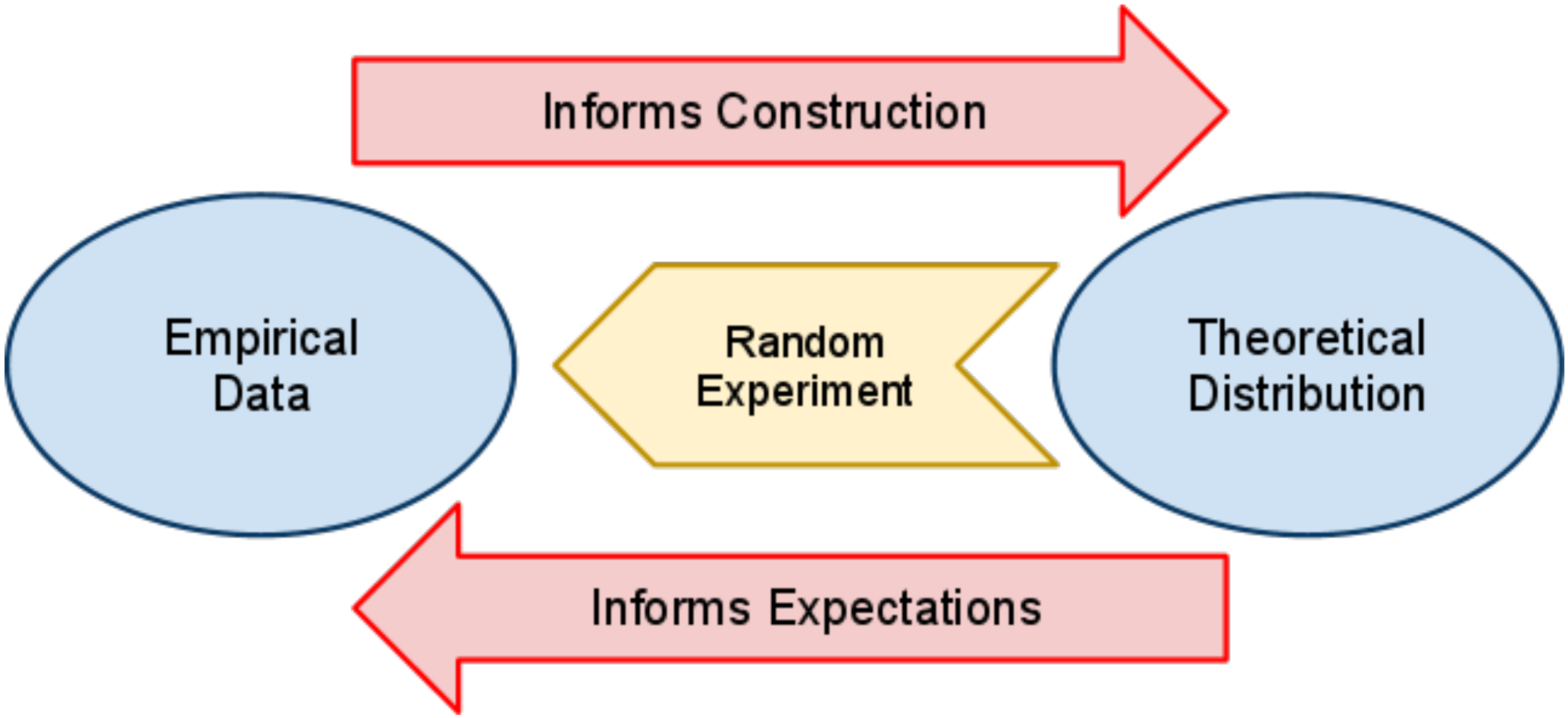
Making Connections as Foundational for Statistical Reasoning

Bidirectional Reasoning



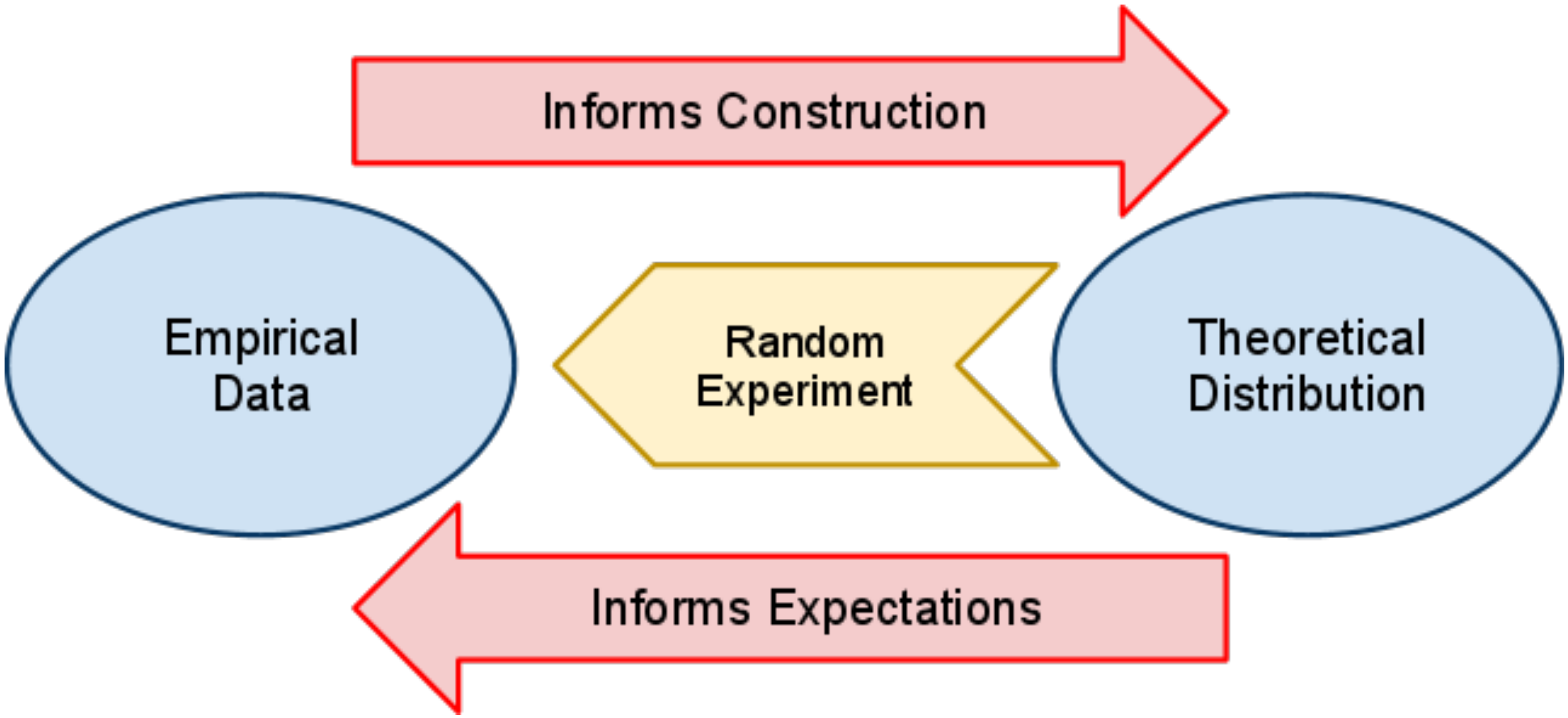
Understanding How Models are Used to Simulate Random Phenomena

Bidirectional Reasoning



Understanding how Data are Produced to Estimate Probabilities

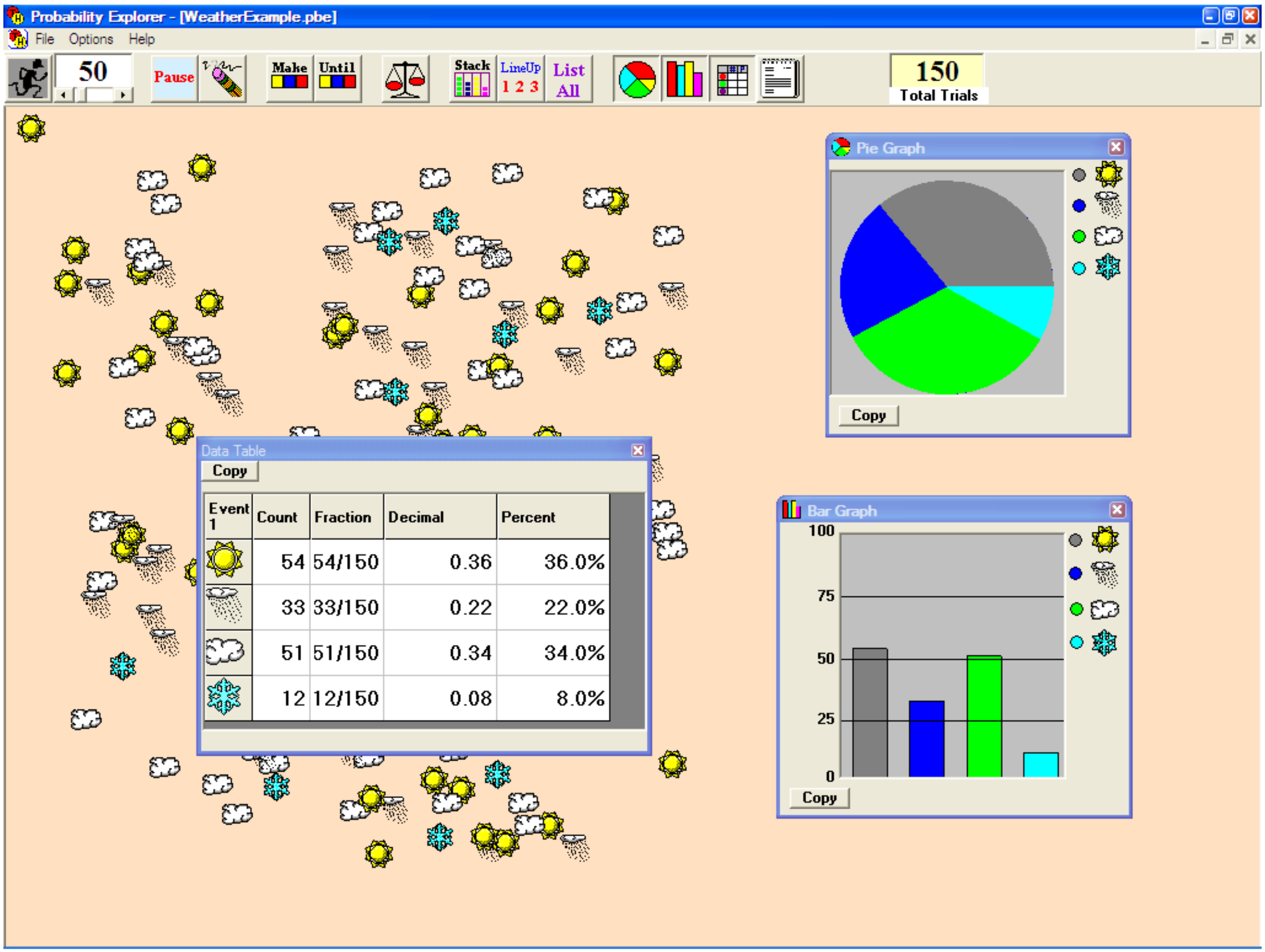
Bidirectional Reasoning



Recognizing How, When and Why
Existing Inferential Tools can be Used

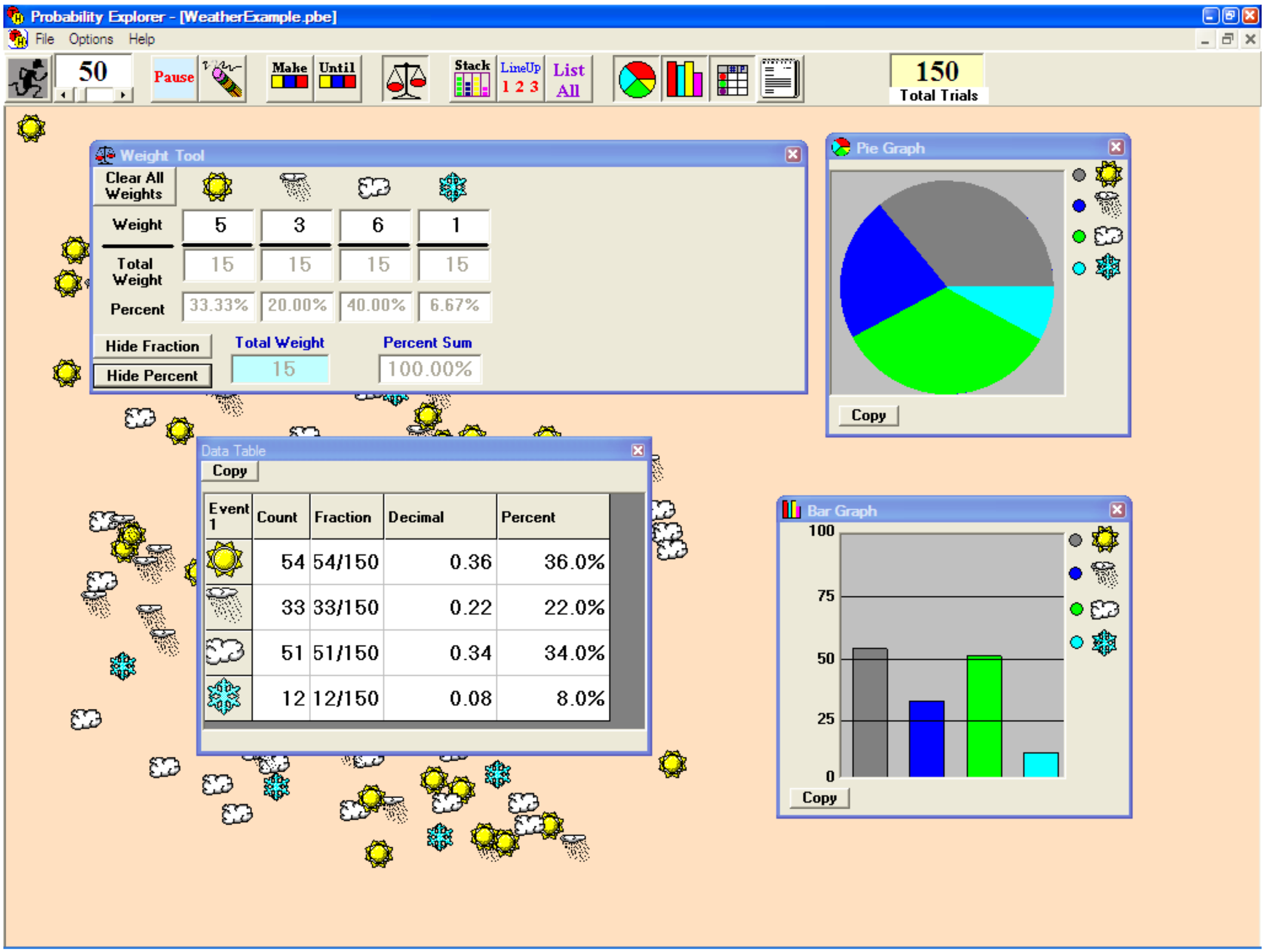
Reasoning by Students ages 9-11

Probability Explorer Simulation Environment

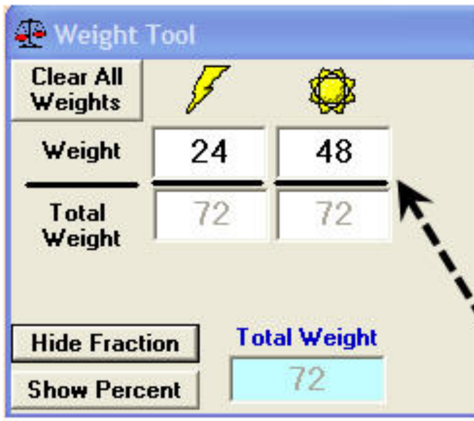


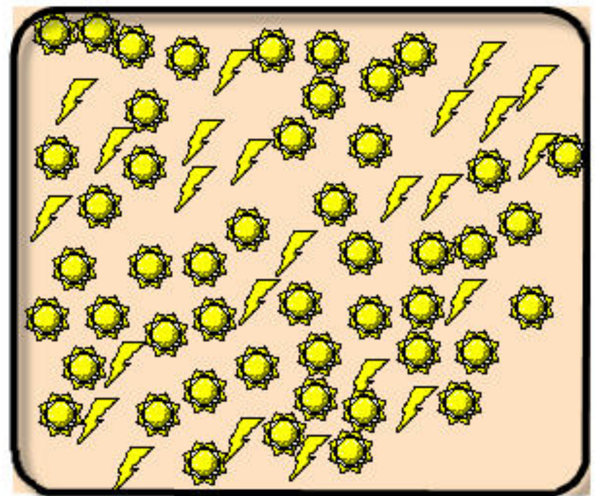
Reasoning by Students ages 9-11

Probability Explorer Simulation Environment



Interpreting a Probability Distribution

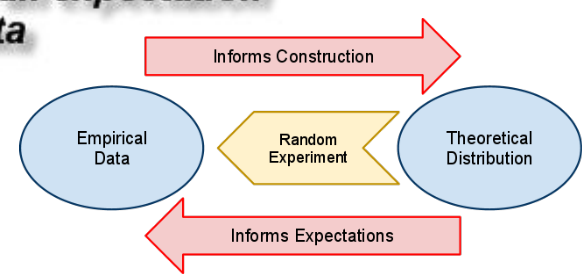




Teacher: 48 and 24. So what do these numbers mean here? 48 over 72?
Jasmine: Forty-eight over 72 …Oh, there are 72 suns and lightening bolts put in the box. Forty-eight of them are suns. Twenty-four of them are lightening bolts. And children put in that many because they think out of 72 days … there are going to be 48 sunny days and 24 thundering days.

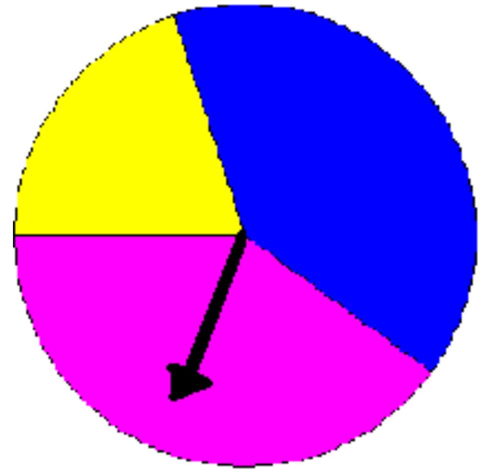
imagining

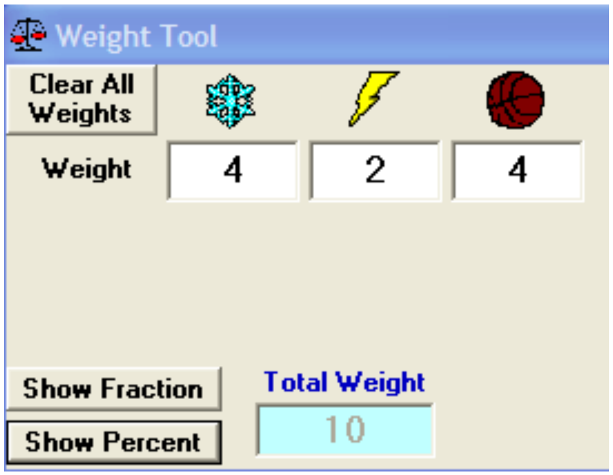
Also an expectation of data

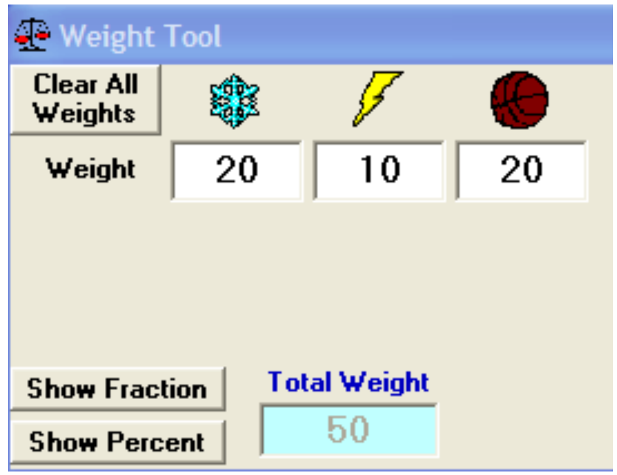


Designing a Model

Original spinner used for visual comparison

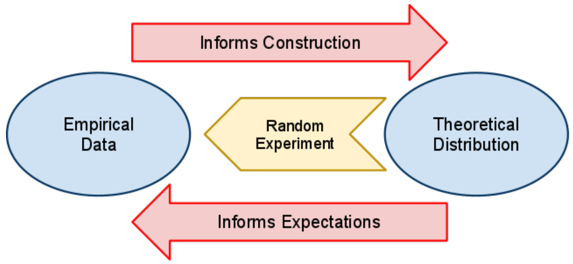




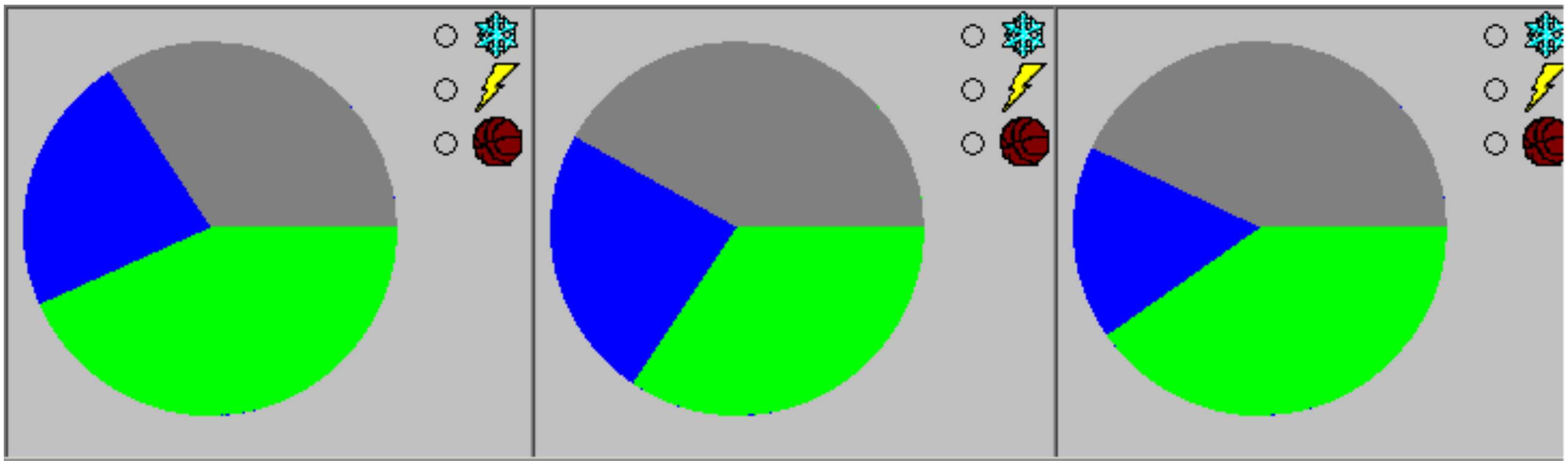


Brandon thought blue and pink were each 40% and that yellow would be 20% since "two yellow areas would make a pink area."

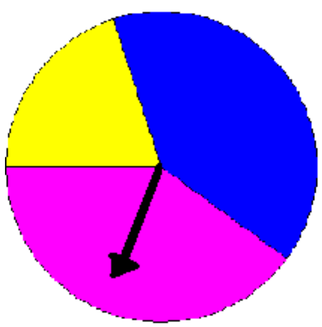
Brandon: "that's not right"
Manuel:"I bet you a billion dollars it is."



Brandon runs trials to test 20:10:20

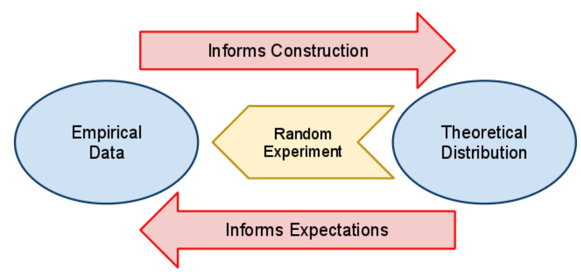


100 trials (34%:23%:43%)
"ok that's right"



50 trials (42%:24%: 34%)
"ah-oh, ah-oh, ah-oh…well that is pretty close….well he's right cause I see the pie graph, I agree with him."

100 trials (43%: 17%:40%)
"dang, that is soooo exact"

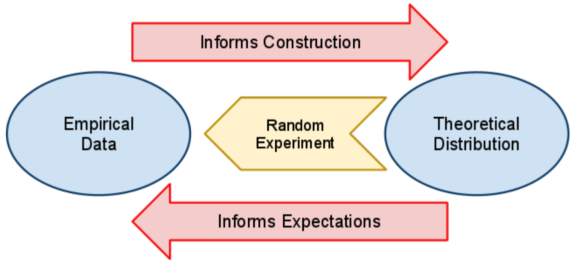


Let’s consider instructional tasks

Two Probability Questions

- Assume a coin is “fair”. If we toss the coin five times, how many heads will we get?
- You pick up a coin. Is this a fair coin?

From GAISE K-12 report (Franklin et al., 2005)

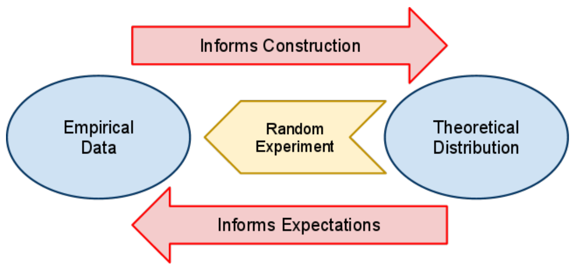


Two Probability Questions

- Assume a coin is “fair”. If we toss the coin five times, how many heads will we get?

- You pick up a coin. Is this a fair coin?
  Answer the first question without assuming the coin is fair. How sure are you of your answer?

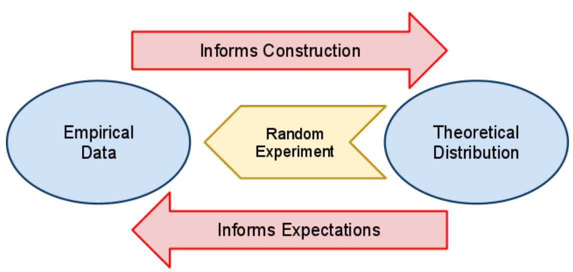
From GAISE K-12 report (Franklin et al., 2005)



Probability in Instruction

Most Common:
Mathematical probability and construction of distributions via classical approaches that don't use data

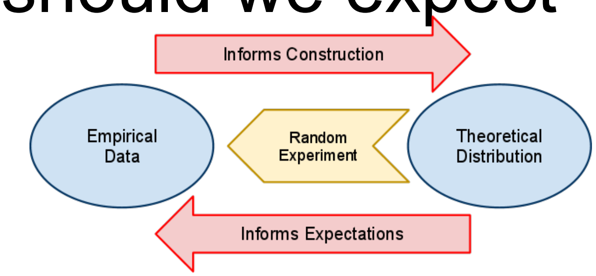
Ex:
What is the probability of getting a green ball from an urn with 2 green and 3 red balls?



Probability in Instruction

Many "Enlightened" Classrooms:
Simple bidirectional reasoning: Expectations that empirical proportions will have high variance. Larger samples may have proportions that show less variance from theoretical, and thus better.
(Issues with Instructional Enactment)

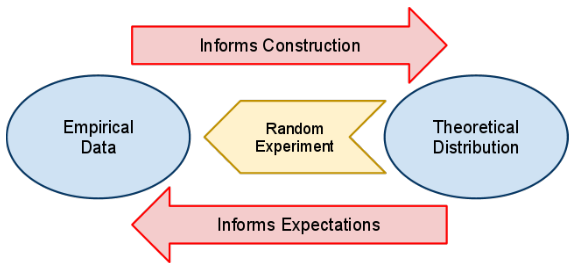
Ex:
We have a theoretical probability of 2/5's for drawing a green ball from the urn. Each student is to draw with replacement 20 balls from their urn and note the percentage that is green. How many balls should we expect to be green? Is that how many you got?



Probability in Instruction

Making Better Headway:
Better bidirectional reasoning, but still mostly from distribution to data: Explore aspects of sample variation, due mostly to sample size, from given distributions.

Ex:
Taking samples of size 4 (with replacement), do we get very many samples with no green balls? How about samples of 20?

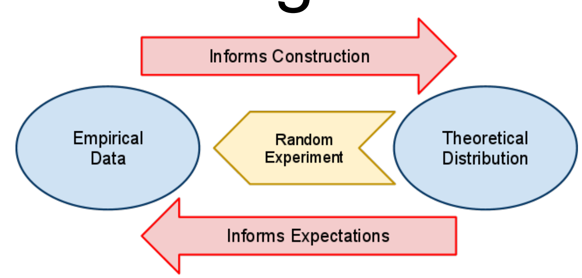


Probability in Instruction

Coming on the Scene:
Using data to make informal inferences about a distribution that is not completely given. Exploring distributions that are not uniform or combinatorial.

Ex:
Given an urn, what proportion of the balls are green? Are there any orange balls in the urn? Are you sure?

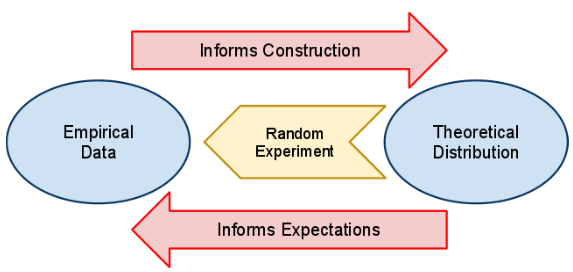
A local company is making dice that are supposed to be fair. How many times should they test their dice to be really sure that they are fair? Would you recommend using the dice from this company?



Questions

Does axiomatic or combinatorial probability instruction promote reasoning about probabilistic phenomena of the nature desired for statistical reasoning and thinking?

In the given definitions of reasoning and thinking, does use of the word “understanding” differentiate well enough between procedural and conceptual understanding?



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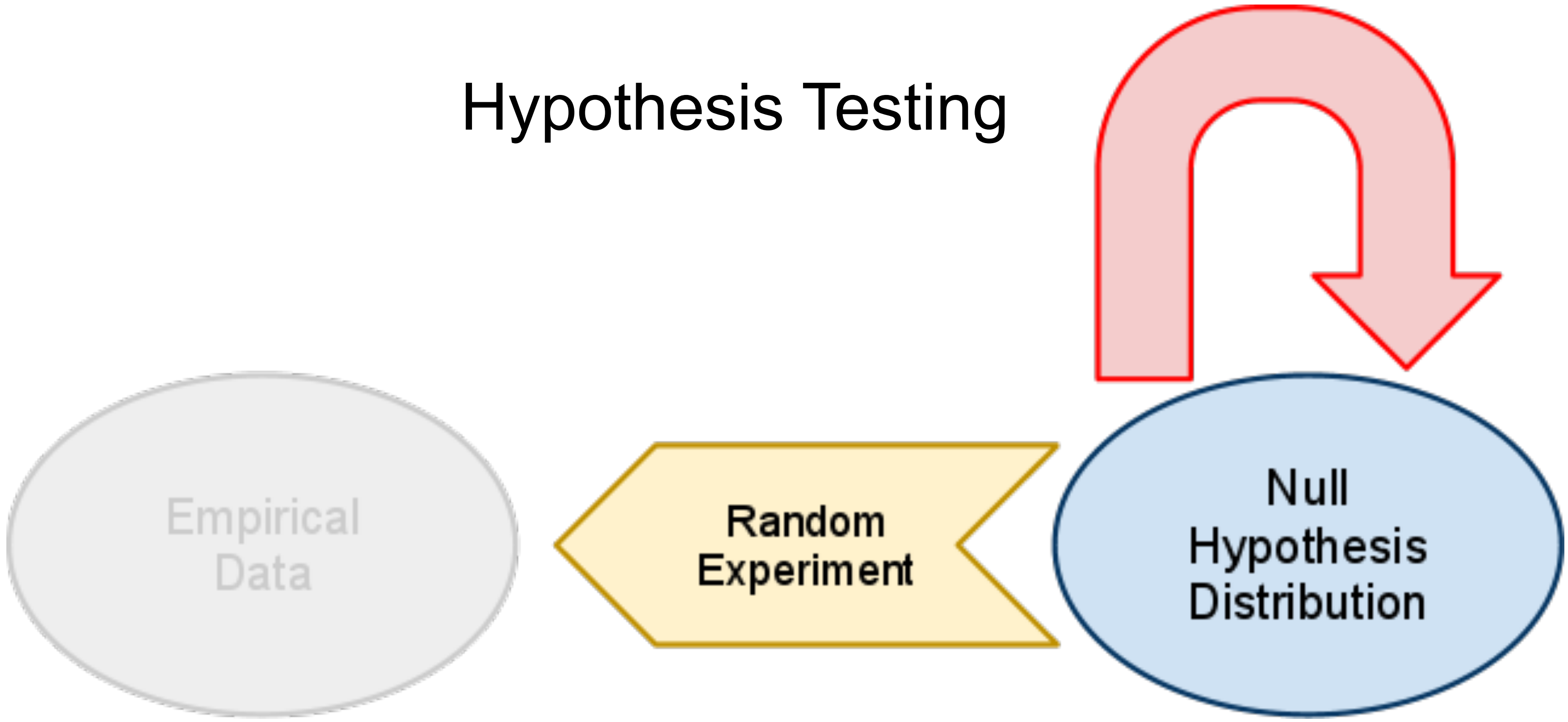
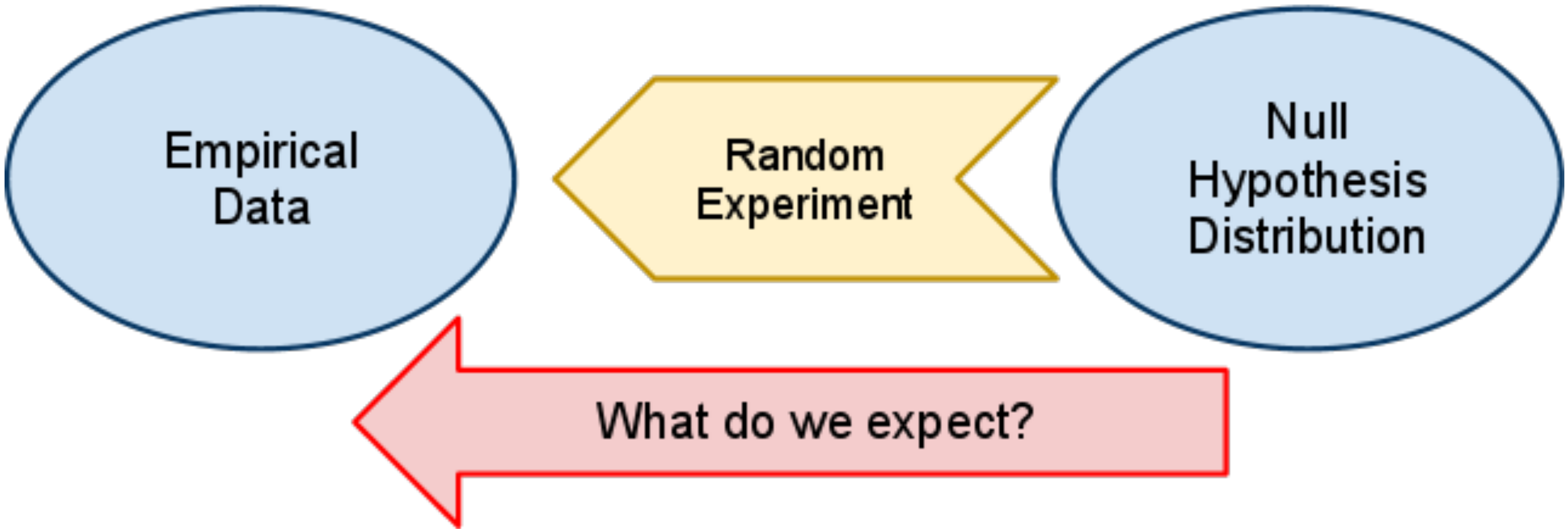
Hypothesis Testing

Hypothesis Testing



Hypothesis Testing



Hypothesis Testing