Sample an AIMS activity: What Makes the Standard Deviation Larger or Smaller (NSF DUE-0535912)

Bob delMas, Joan Garfield, and Andy Zieffler University of Minnesota

Overview of Webinar

- Goals of AIMS
- Materials developed
- Variability: Why is it important to understand?
- Learning sequence for understanding variability
- An example activity: What Makes the Standard Deviation Larger or Smaller?



Goals of AIMS

- Integrate and adapt innovative materials developed for introductory statistics
- Develop lesson plans and activities for important topics
- Focus on developing statistical literacy and reasoning (see GAISE; <u>http://www.amstat.org/education/gaise/</u>)
- Build materials on important instructional design principles



Materials Developed

- AIMS website (<u>http://www.tc.umn.edu/~aims/</u>)
- Lesson plans (28)
- 1 to 3 Activities for each lesson
- Suggested sequences of activities
- Compilation of research:

Developing Students' Statistical Reasoning: Connecting Research and Teaching Practice by Joan Garfield and Dani Ben-Zvi

The Importance of Variability

Variability is ... the essence of statistics as a discipline and it is not best understood by lecture. It must be experienced (Cobb, 1992) Understanding Variability is

- Key component of understanding distribution
- Core component of statistical thinking
- Essential for making statistical inferences (Garfield & Ben-Zvi, 2008)

What is Variability?

- Recognize that values can vary or change
- A measure of spread tied to ideas of
 - Center
 - Distribution
- Different sources of variability:
 - Natural variation
 - Measurement error
 - Sampling



What is the Standard Deviation?

- As a concept, tied to the idea of center
- A measure of the extent to which values deviate from center
- Standard deviation as average distance from center
- Coordination of ideas of deviation from the mean (distance) and density (frequency)



Building an Understanding of Variability

Informal Ideas of Variability (Garfield & Ben-Zvi, 2008)

- Data vary
- Results from random processes
- Examine and compare graphs to develop ideas about spread in distributions
- Range as a simple measure of spread



Building an Understanding of Variability

Formal Ideas of Variability (Garfield & Ben-Zvi, 2008)

- Two types: Diversity and Measurement Error
- Comparisons: A lot and a little variability
- Standard deviation as average deviation from the mean
- Factors that effect the the standard deviation
- Representing center and spread in graphs



What Makes the SD Larger or Smaller?

Activity Goals

- Informally estimate "typical" deviation from the mean.
- Understand standard deviation as a measure of spread.
- Understand what makes standard deviations larger or smaller
- Identify distribution with larger standard deviation by comparing graphs.



Activity to Build Prior Knowledge

Post-It Note Dot Plots Activity

- Number line that represents ages of students
- Arrange Post-It notes to meet criteria:
 - Mean age = 21 by placing all notes at 21
 - Move one note to 24. Move 2^{nd} so mean = 21
 - Move one note to 17 and others so mean = 21
- Compute deviations from the mean
 - Use change in deviations for note placement



Standard Deviation Activity

Learn method to determine "average deviation from the mean"



Draw in each deviation from the mean. Estimate the length of the average deviation. Draw a line of this length below the graph.



Standard Deviation Activity

Learn method to determine "average deviation from the mean"



Draw in each deviation from the mean. Estimate the length of the average deviation. Draw a line of this length below the graph.





Activity: Comparing Standard Deviations

Below, you will find five pairs of graphs. The mean for each graph (m) is given just above each histogram. For each pair of graphs presented,

- Indicate which one of the graphs has a larger standard deviation or if the two graphs have the same standard deviation.
- Explain why. (Hint: Try to identify the characteristics of the graphs that make the standard deviation larger or smaller.)









- Both graphs have the same range
- Relatively few deviations of -2 or 2 in Graph A
- Majority of deviations are 2 or -2 in Graph B







- One graph is the mirror image of the other
- Frequencies of each mean deviation the same
- Therefore, average deviation the same









- All deviations in Graph B are -2.5 or 2.5
- 1/3 deviations in Graph A are -2.5 or 2.5
- Therefore, average deviation larger for Graph B

Understanding of Standard Deviation

Guide whole-class discussion of explanations:

- Larger Range is not necessarily Larger SD
- Same Range is not necessarily equal SD
- Larger number of possible values ≠ Larger SD
- Evenly spread out ≠ Larger SD
- Majority of deviations close to mean = Smaller SD
- Majority of deviations far from mean = Larger SD



Further Development of Understanding

Building on Formal Ideas of Variability

- Factors that affect range and IQR
- Use center and spread to compare groups
- Roles of variability within and between groups when comparing groups
- Relationship between sampling variation and sample size
- Variability in bivariate plots

University of Minnesota

AIMS Resources

- AIMS website (<u>http://www.tc.umn.edu/~aims/</u>)
- Lesson and lesson plans
- Sequence of ideas about variability
- Technology tools used
- Link to the new book by Garfield and Ben-Zvi (provides research foundations for lessons)



REFERENCES

Cobb, G. (1992), Teaching statistics. In L. Steen (Ed.), Heeding the call for change: Suggestions for curricular action, MAA Notes, Vol. 22, 3-33.

Garfield, J. & Ben-Zvi, D. (2008). *Developing Students' Statistical Reasoning: Connecting Research and Teaching Practice*. Springer.



Thank You!

• Please check out and use our materials.

AIMS website (<u>http://www.tc.umn.edu/~aims/</u>)

Please send us your feedback.
Joan Garfield: jbg@umn.edu
Bob delMas: delma001@umn.edu
Andy Zieffler: zief0002@umn.edu

