Using Cognitive Task Analysis to Uncover Misconceptions in Statistical Inference Courses

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Hi, my name is Mikaela Meyer, and I'm a Ph.D. student in the Teaching Statistics research group in the Department of Statistics and Data Science at Carnegie Mellon University.

Why do students struggle with introductory statistical inference problems?

- Our sophomore statistical inference course draws a large, diverse audience
- Difficult to tell how best to reach this audience or what skills to have them practice

Think-aloud interviews:

- Developed in cognitive science by Ericsson and Simon
- Interviewees read the question aloud and narrate their thinking
- No feedback from the interviewer (verbal reactions, facial expressions, ...)

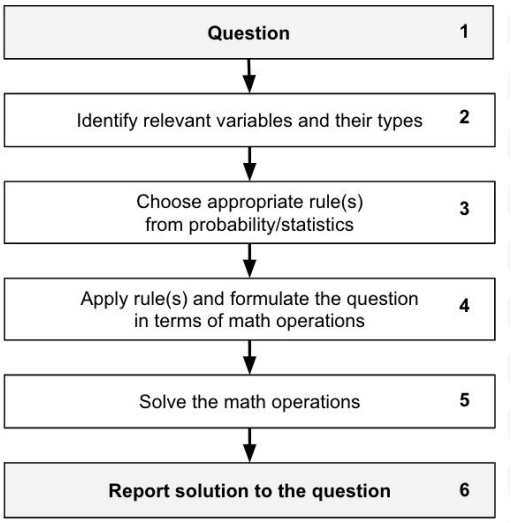
At Carnegie Mellon, our sophomore-level statistical inference course covers topics like maximum likelihood estimation, hypothesis testing, and confidence intervals, and has nearly 300 students from many majors. All of these students have taken a probability course, but they have widely varying mathematical and statistical backgrounds. Many of these students struggle to complete typical inference problems. Why is this? We wanted to identify specific skills they might lack, which would help *us* learn how to help *students* learn better.

To isolate the skills involved in solving inference problems, we decided to conduct think aloud interviews and use cognitive task analysis to identify steps where students struggled.

Think aloud interviews are a cognitive science tool where respondents are asked to narrate everything they are thinking about while answering a question. Rather than coming to the answer and explaining afterwards how that answer was reached, participants in think aloud interviews narrate the steps they are taking *as they take them*.

Cognitive Task Analysis breaks problems into steps so we can diagnose specific skills with which students struggle

- Interviews with experts reveal the skills needed to complete the problems
- Interviews with students reveal the *specific* skills they stumble on

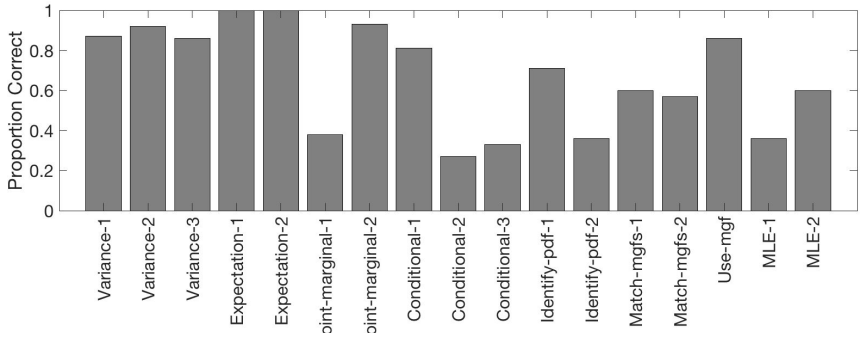


Cognitive task analysis identifies the steps that experts and novices use to navigate from problem statement to solution. These steps are often implicit, and experts do them automatically without realizing it; we used think-aloud interviews to identify these steps explicitly.

Based on discussions with experienced course instructors, we focused on problems requiring students to identify which variable is relevant to a problem: for example, which variable must be differentiated with respect to. Instructors suspected students struggle with this skill but are never taught it explicitly. Think-aloud interviews with students allowed us to test this hypothesis and determine which specific cognitive tasks are most difficult for them.

We interviewed experts and novices to assess and validate our CTA steps

- Interviewed sixteen undergraduate students and eight Ph.D. students for 60 minutes each
- Each answered at most 25 questions, from calculating the expectation of a random variable to finding a conditional distribution given joint and marginal



First, we created about 25 problems based on topics typically covered in our introductory statistical inference course (such as finding the marginal density when given the joint density or finding the maximum likelihood estimator for some distribution). We also included some basic mathematical and probability questions, in case students' unfamiliarity with mathematics was preventing them from solving statistics problems.

Then we began conducting think aloud interviews with paid volunteers. We interviewed 8 statistics PhD students as "experts" to identify the cognitive tasks used in correctly solving the problems.

Finally, we interviewed 16 undergraduate students currently taking the introduction to statistical inference course to see if the steps they were taking to solve the problem followed our outline. We timed the interviews so they should already have learned most of the topics, but as you can see, some questions were harder than others. By seeing which parts of the question they struggled with the most, we could identify the specific cognitive tasks we could try to teach better.

X and Y are random variables with joint density

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{4}(9 - 6x - 6y + 4xy) & 0 \le x, y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find $f_X(x)$, the marginal density of X.

What experts said…

- "So now I have to remember how to do calculus, which is too bad"
- "I know the limits of integration, but never do integration by hand anymore."
- "I always imagine this in terms of two-way tables as I decide what variable to sum, in this case integrate because it is continuous…"

What novices said…

- "I think we just derive it with respect to *X*"
- "I am pretty sure that's when you take the integral and if we wanted *x* we take the integral over *y* for some reason, we just do it opposite, gotta be a goof for no reason"
- Generally took time to integrate and reason through the set up

Here is one of the questions that we asked during our interviews. We asked them to find the marginal distribution of x, given the joint distribution of x and y.

Of the 8 experts who answered this question, only one made a math mistake and arrived at the incorrect answer; the others all answered the problem correctly, though they were not excited to have to integrate by hand. The experts not only knew what approach they needed to take to solve the problem, but they seemed to understand why this approach is correct.

Among the novices, only 6 of 16 arrived at the correct answer -- and three of those arrived at the correct answer despite misreading the bound's notation. Four of the remaining students also struggled to interpret the bounds. The other mistakes involved selecting the right operations to solve the problem -- for example, six students tried to use a derivative instead of an integral. Even among students with a correct written answer, we observed during the interviews that many took a long time to think through the problem setup and the choice of method to solve it, showing that the topic was not conceptually clear to them.

In summary, we see that experts seemed to know what to do to solve this problem and why to do it almost immediately, while novices did not always know what to do automatically and did not seem to know why they needed to take the steps they took.

Conclusions

- Students struggle to identify what operations to apply to which variables
- Instructors can use the CTA steps as a basis for improving instruction and writing targeted assessments
- Research suggests that identifying learning components and designing appropriate instruction improves learning (Koedinger et al., 2012)

This is just one example of how interviews and CTAs help us identify specific reasons students struggle with questions. We can think of these CTA steps as topics to target when teaching a concept and adapt our instruction styles to better teach these specific steps. Prior research (Koedinger) suggests that instruction and practice about specific CTA steps can transfer well to more complicated problems because students have a better understanding of how to approach tasks generally.

Similarly, when we write homework assignments or exams, we can choose to target questions to specific cognitive tasks: we can break down the problems into asking “what definition or rule do you use here”, “set up the correct integral”, etc. This will help better assess student learning and help scaffold their learning of the skills.

Thank you!

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http://www.stat.cmu.edu/teachstat/

We're now exploring the rest of our results and their implications for teaching statistical inference, and we plan to use think-aloud interviews and CTAs on other statistical topics as well. Check out our website for updates on our research and results from other think-aloud studies. Thank you.

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